ELECTROMAGNETICS – I SECOND MIDTERM EXAM

Dr. Salih FADIL

April 29, 2009

#1) Point charges $5\,\mu C$, $-3\,\mu C$, $2\,\mu C$ and $10\,\mu C$ are located at (-12, 0, 5), (0, 3, -4), (2, -6, 3) and (3, 0, 0) respectively. Calculate the electric flux through the spherical surfaces at:

a) $r = 1$
b) $r = 10$
c) $r = 15$.

#2) The line $y = 1, z = -3$ carries charge 30 *nC/m,* while the plane $x = 1$ carries a charge $20\,nC/m^2$. Find $\vec{E}$ at the origin.

#3) Two point charges $Q_1 = 3\,nC$ and $Q_2 = -2\,nC$ are placed at $(0, 0, 0)$ and $(0, 0, -1)$ respectively. Assuming zero potential at infinity, find the potential at:

a) $(0, 1, 0)$
b) $(1, 1, 1)$

ELECTROMAGNETICS-I SECOND MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

April 29, 2009

#1)

Point charge	Location (x, y, z)	r
5 μC	(-12, 0, 5)	$[(12)^2+25]^{1/2}=13$
-3 μC	(0, 3, -4)	$[9+16]^{1/2}=5$
2 μC	(2, -6, 3)	$[4+36+9]^{1/2}=7$
10 μC	(3, 0, 0)	$[9+0+0]^{1/2}=3$

Gauss equation $\Psi = \oint_S \vec{D}\cdot d\vec{S} = Q_{enc}$

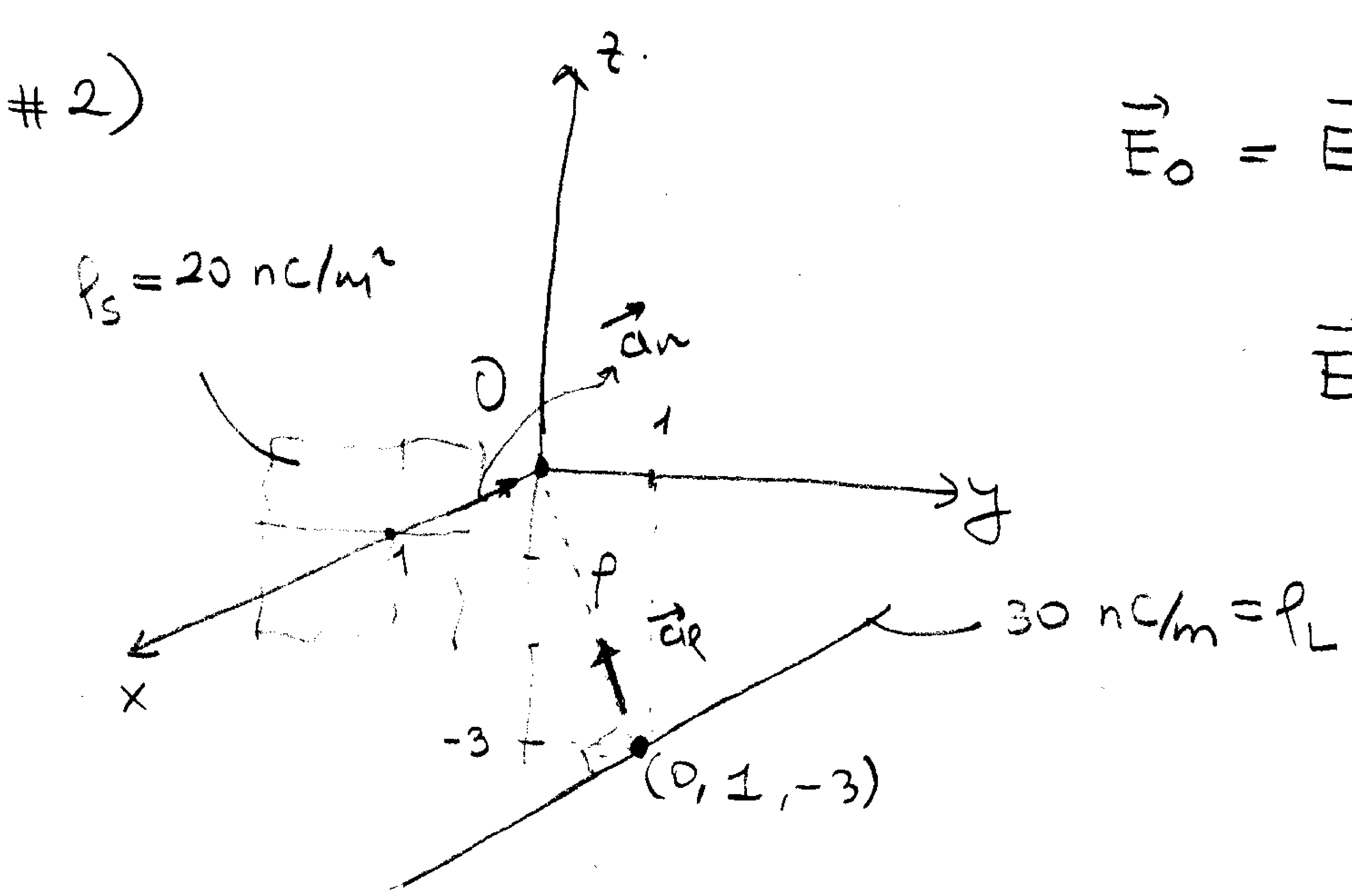
a) $\Psi = 0\ \mu C$, for $r=1$ surface

b) $\Psi = -3+2+10 = 9\ \mu C$, for $r=10$ surface

c) $\Psi = 5-3+2+10 = 17-3 = 14\ \mu C$, for $r=15$ surface

This is an extremely easy question... ☺

#2)



$$\vec{E}_O = \vec{E}_{\rho_L} + \vec{E}_{\rho_s}$$

$$\vec{E}_{\rho_L} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

$$\vec{E}_{\rho_s} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n$$

$\rho = \sqrt{9+1} = \sqrt{10}$

$$\vec{a}_\rho = \frac{(0,0,0)-(0,1,-3)}{\sqrt{10}} = \frac{1}{\sqrt{10}}\left(-\vec{a}_y + 3\,\vec{a}_z\right)$$

$$\vec{E}_{\rho_L} = \frac{30\,10^{-9}}{2\pi \frac{10^{-9}}{36\pi}\sqrt{10}}\,\frac{1}{\sqrt{10}}\left(-\vec{a}_y + 3\vec{a}_z\right) = \frac{18\times 30}{10}\left(-\vec{a}_y + 3\vec{a}_z\right)$$

18

$$\vec{E}_{\rho_L} = -54\,\vec{a}_y + 162\,\vec{a}_z \text{ V/m.}$$

$\vec{a}_n = -\vec{a}_x$ (from source to field)

$$\vec{E}_{\rho_s} = \frac{20\,10^{-9}}{2\,\frac{10^{-9}}{36\pi}}(-\vec{a}_x) = -360\pi\,\vec{a}_x = -1131\,\vec{a}_x$$

10

$$\boxed{\vec{E}_o = -1131\,\vec{a}_x - 54\,\vec{a}_y + 162\,\vec{a}_z \text{ V/m}}$$

#3) $Q_1 = 3$ nC at $(0,0,0)$

$Q_2 = -2$ nC at $(0,0,-1)$

assuming zero potential at infinity (absolute poten.)

a) Potential at $(0,1,0)$; $V(r) = \dfrac{Q}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|}$ $\vec{r}$: pos. vec. of field, $\vec{r}\,'$: " " " source

$$V(0,1,0) = \frac{3\,10^{-9}}{4\pi \frac{10^{-9}}{36\pi}\,\underbrace{|(0,1,0)-(0,0,0)|}_{(0,1,0)}} + \frac{-2\,10^{-9}}{4\pi \frac{10^{-9}}{36\pi}\,\underbrace{|(0,1,0)-(0,0,-1)|}_{(0,1,1)}}$$

9 9

$$V(0,1,0) = \frac{27}{1} + \frac{-18}{\sqrt{2}} = \underline{\underline{14.272}} \text{ volt}$$

b) $$V(1,1,1) = \frac{3\,10^{-9}}{4\pi \frac{10^{-9}}{36\pi}\,\underbrace{|(1,1,1)-(0,0,0)|}_{(1,1,1)}} + \frac{-2\,10^{-9}}{4\pi \frac{10^{-9}}{36\pi}\,\underbrace{|(1,1,1)-(0,0,-1)|}_{(1,1,2)}}$$

9 9

$$= \frac{27}{\sqrt{3}} - \frac{18}{\sqrt{6}} = 8.24 \text{ volt}$$