ELECTROMAGNETICS-I FIRST MIDTERM EXAM

Dr. Salih FADIL — July 13, 2012

#1) Show that vectors $\vec{a} = (4, 0, -1)$, $\vec{b} = (1, 3, 4)$ and $\vec{c} = (-5, -3, -3)$ form the sides a triangle. Is this a right angled triangle? Calculate the area of the triangle. Use <u>only</u> vector algebra in your solution.

#2) The scalar component of a vector $\vec{A}$ along $\vec{B} = 3\vec{a}_x - 4\vec{a}_z$ is 10. If $\vec{A}$ has the same direction with $\vec{C} = \vec{a}_x + 2\vec{a}_y + \vec{a}_z$, determine $\vec{A}$.

#3) Let $\vec{A} = \rho\cos\phi\,\vec{a}_\rho + z\sin\phi\,\vec{a}_\phi - \rho z^2\,\vec{a}_z$ and

$$\vec{B} = r\sin\theta\,\vec{a}_r + r^2\cos\theta\sin\phi\,\vec{a}_\theta$$

Find the unit vector, in cartesian coordinates, which is perpendicular both $\vec{A}$ and $\vec{B}$ in $P(3, -2, 6)$.

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SOLUTION MANUAL

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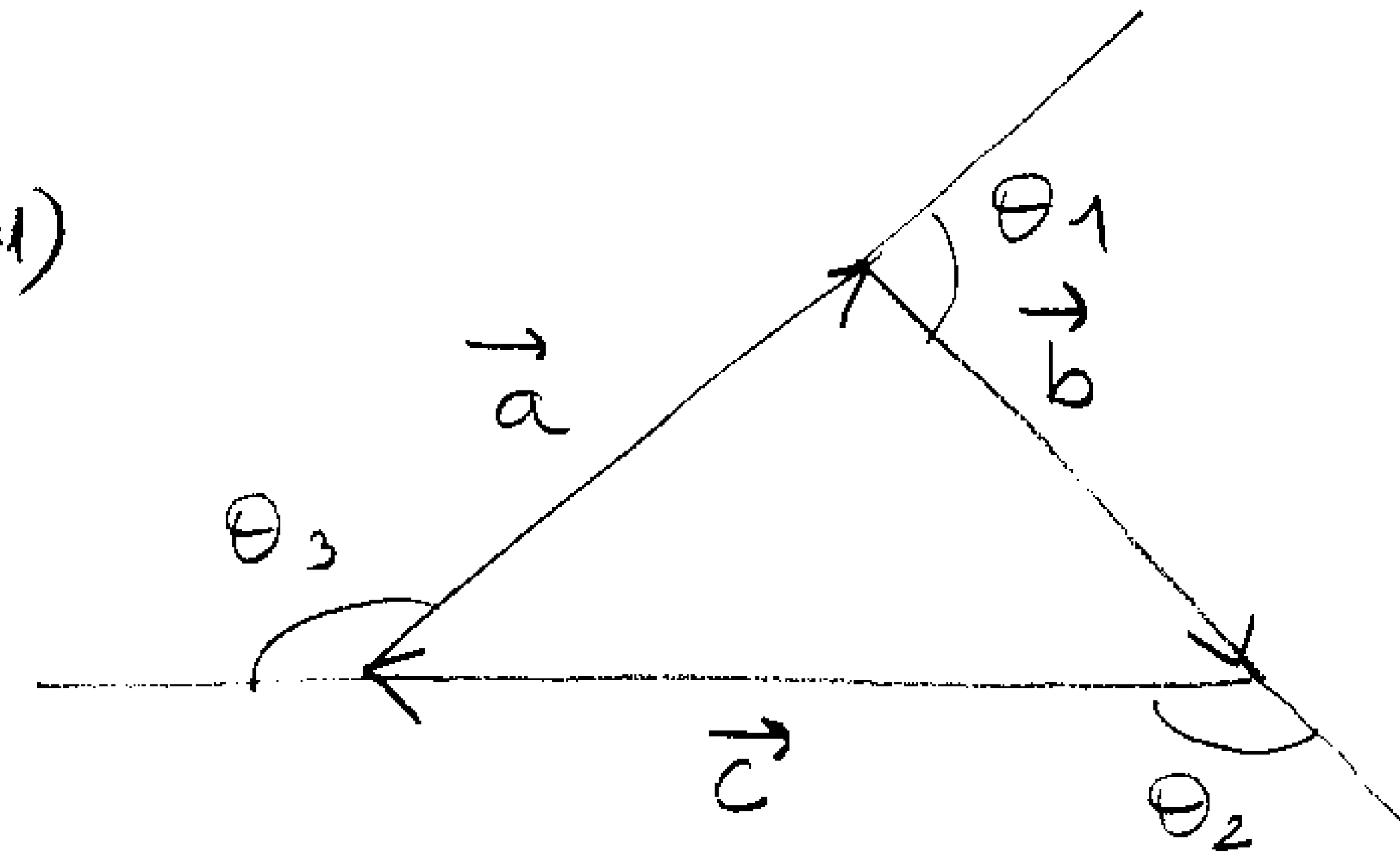
#1)



if $\vec{a}+\vec{b}+\vec{c}=0$ then $\vec{a}$, $\vec{b}$ and $\vec{c}$ form three sides of a triangle. So

$$\vec{a}+\vec{b}+\vec{c} = \cancel{4\vec{a}_x} - \cancel{\vec{a}_z} + \cancel{\vec{a}_x} + \cancel{3\vec{a}_y} + \cancel{4\vec{a}_z} - \cancel{5\vec{a}_x} - \cancel{3\vec{a}_y} - \cancel{3\vec{a}_z} = 0$$

This is true they form three sides of a triangle. 1

If the triangle is a right-angled triangle, one of angles $\theta_1, \theta_2, \theta_3$ must be 90°, or

$$\vec{a} \perp \vec{b} \text{ or } \vec{b} \perp \vec{c} \text{ or } \vec{c} \perp \vec{a}$$

$$\vec{a}\cdot\vec{b} = 0 \text{ or } \vec{b}\cdot\vec{c} = 0 \text{ or } \vec{c}\cdot\vec{a} = 0$$

$\vec{a}\cdot\vec{b} = 4-4=0 \qquad \theta_1 = 90^\circ$

$\vec{b}\cdot\vec{c} = -5-9-12 \neq 0$ 1

$\vec{c}\cdot\vec{a} = -20+3 \neq 0$

So, it is a right-angled triangle

$$\text{Area} = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|\vec{b}\times\vec{c}| = \frac{1}{2}|\vec{c}\times\vec{a}|$$

1

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 4 & 0 & -1 \\ 1 & 3 & 4 \end{vmatrix} = \vec{a}_x(0+3) - \vec{a}_y(16+1) + \vec{a}_z(12-0)$$

$$\text{Area} = \frac{1}{2}\left[3^2+17^2+12^2\right]^{1/2} = 10.51 \ (\text{unit})^2$$

#2) $\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$, $\vec{C} = \vec{a}_x + 2\vec{a}_y + \vec{a}_z$

$\vec{B} = 3\vec{a}_x - 4\vec{a}_z$

$$\vec{A}\cdot\vec{B} = A\, B \cos\theta_{AB} = A_B \cdot B \qquad A_B = \frac{\vec{A}\cdot\vec{B}}{B} = \frac{3A_x - 4A_z}{\sqrt{9+16}}$$

$$A_B = \frac{1}{5}(3A_x - 4A_z) = 10$$

if $\vec{A}$ has the same direction with $\vec{C}$, then.

$\vec{A} = k\vec{C}$, $k>0$

$A_x = k$ | $3A_x - 4A_z = 50$

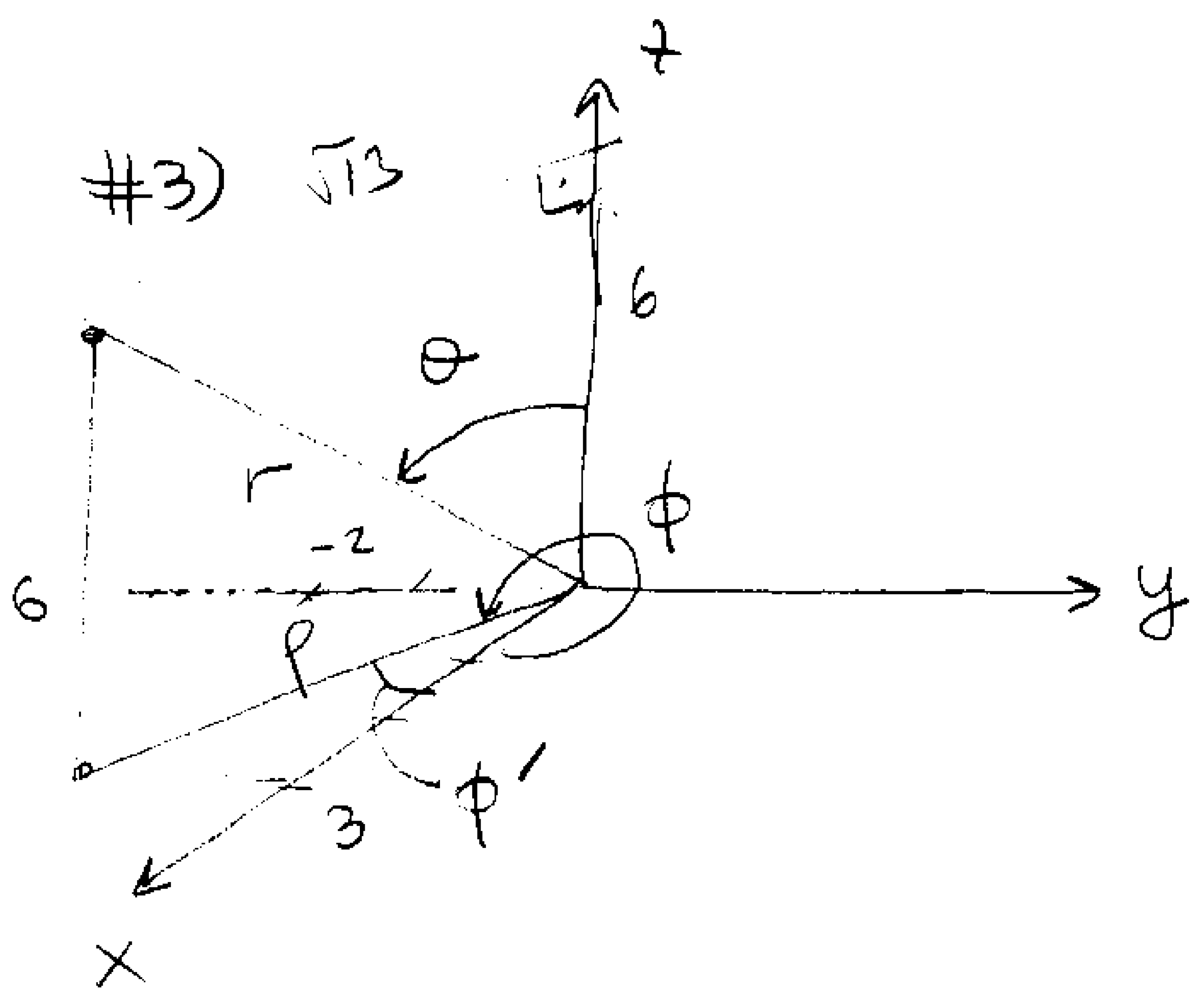
$A_y = 2k$ | $3k - 4k = 50$ $\boxed{k = -50}$

$A_z = k$

Actually, according to the obtained result, vector $\vec{A}$ and $\vec{C}$ have opposite directions since $k<0$.

$$\vec{A} = -50(\vec{a}_x + 2\vec{a}_y + \vec{a}_z)$$

#3) $\sqrt{13}$



$\phi' = \tan^{-1}\left(\frac{2}{3}\right) = 33,69°$

$\phi = 360° - 33,69° = 326,3°$

$\rho = \sqrt{9+4} = \sqrt{13}$

$z = 6$

$\tan^{-1}\frac{\sqrt{13}}{6} = \theta$, $\theta = 31°$

$r = \sqrt{13+36} = 7$

$$A_\rho = \sqrt{13}\cos(326{,}3^\circ) = 2{,}996$$

$$A_\phi = 6\,\sin(326.3^\circ) = -3.329$$

$$A_z = -\sqrt{13}\cdot 36 = -129{,}8$$

$$B_r = 7\cdot\sin(31^\circ) = 3{,}6052$$

$$B_\theta = 49\cdot\cos(31^\circ)\cdot\sin(326{,}3^\circ) = -23{,}304$$

$$B_\phi = 0$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos(326{,}3^\circ) & -\sin(326{,}3^\circ) & 0 \\ \sin(326{,}3^\circ) & \cos(326{,}3^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.996 \\ -3{,}329 \\ -129.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}83195 & 0{,}55484 & 0 \\ -0{,}55484 & 0{,}83195 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2{,}996 \\ -3{,}329 \\ -129{,}8 \end{bmatrix} = \begin{bmatrix} 0{,}64546 \\ -4.4319 \\ -129{,}8 \end{bmatrix}$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin 31^\circ\cdot\cos(326{,}3^\circ), & \cos 31^\circ\cos(326{,}3^\circ), & -\sin(326{,}3^\circ) \\ \sin 31^\circ\,\sin(326{,}3^\circ) & \cos 31^\circ\,\sin(326{,}3^\circ) & \cos(326{,}3^\circ) \\ \cos 31^\circ & -\sin 31^\circ & 0 \end{bmatrix}\cdot$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 0{,}42848 & 0{,}71312 & +0{,}55484 \\ -0{,}28576 & -0{,}47559 & 0{,}83195 \\ 0{,}85716 & -0{,}51503 & 0 \end{bmatrix} \begin{bmatrix} 3{,}6052 \\ -23{,}304 \\ 0 \end{bmatrix} \cancel{\begin{bmatrix} 3{,}6052 \\ -23{,}304 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} -15.073 \\ 10.0529 \\ 15.092 \end{bmatrix}$$

$$\vec{C} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0,64546 & -4.4319 & -129,8 \\ -15.073 & 10,0529 & 15.092 \end{vmatrix}$$

$$= \vec{a}_x \left(-15.092 \times 4.4319 + 10,0529 \times 129,8\right)$$
$$- \vec{a}_y \left(0,64546 \times 15.092 - 15.073 \times 129,8\right)$$
$$+ \vec{a}_z \left(0,64546 \times 10,0529 - 15.073 \times 4,4319\right)$$
$$= 1237.98\,\vec{a}_x + 1946,74\,\vec{a}_y - 60.313\,\vec{a}_z$$

$$C = \left[(1237,98)^2 + (1946,74)^2 + (60,313)^2\right]^{1/2}$$
$$= 2307.82$$

$$\vec{a}_c = \pm \frac{\vec{C}}{C} = \pm \left(0,53642\,\vec{a}_x + 0,84354\,\vec{a}_y - 0,026134\,\vec{a}_z\right)$$