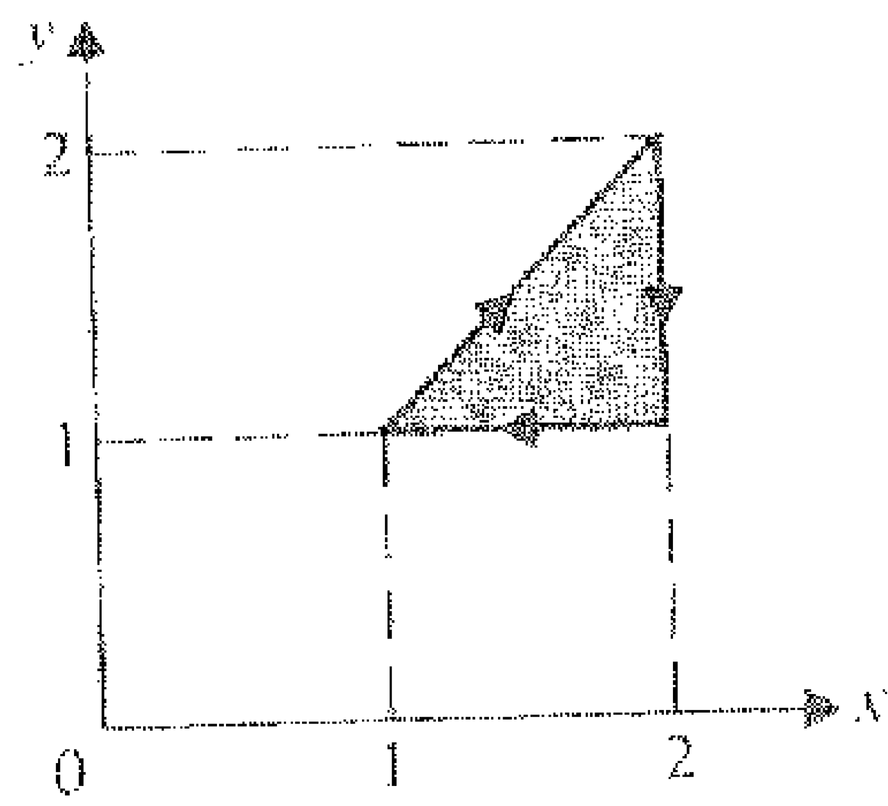
ELECTROMAGNETICS I FIRST MIDTERM EXAM

Dr. Salih FADIL **July 15, 2011**

#1) Consider the vector function $\vec{A} = 3x^2y^3\vec{a}_x - x^3y^2\vec{a}_y$ Calculate $\oint_L \vec{A}\cdot d\vec{l}$ around the triangular loop that is shown in the figure.



#2) A field is expressed in spherical coordinates by $\vec{E} = \dfrac{25}{r^2}\vec{a}_r$.

a) Find $|\vec{E}|$ and E_x at the point $P(-3,4,-5)$.

b) Find the angle that $\vec{E}$ makes with the vector $\vec{B} = 2\vec{a}_x - 2\vec{a}_y + \vec{a}_z$ at point P.

#3) Given the vector field $\vec{H} = \rho\, z \cos\phi\, \vec{a}_\rho + e^{-z}\sin\dfrac{\phi}{2}\vec{a}_\phi + \rho^2\vec{a}_z$. At point $P(1, \pi/3, 0)$ find:

a) $\vec{H} \times \vec{a}_\rho$

b) the vector component of $\vec{H}$ normal to surface $\rho = 1$.

c) The scalar component of $\vec{H}$ tangential to the plane $z = 0$.

GOOD LUCK ...☺

ELECTROMAGNETICS-I FIRST MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

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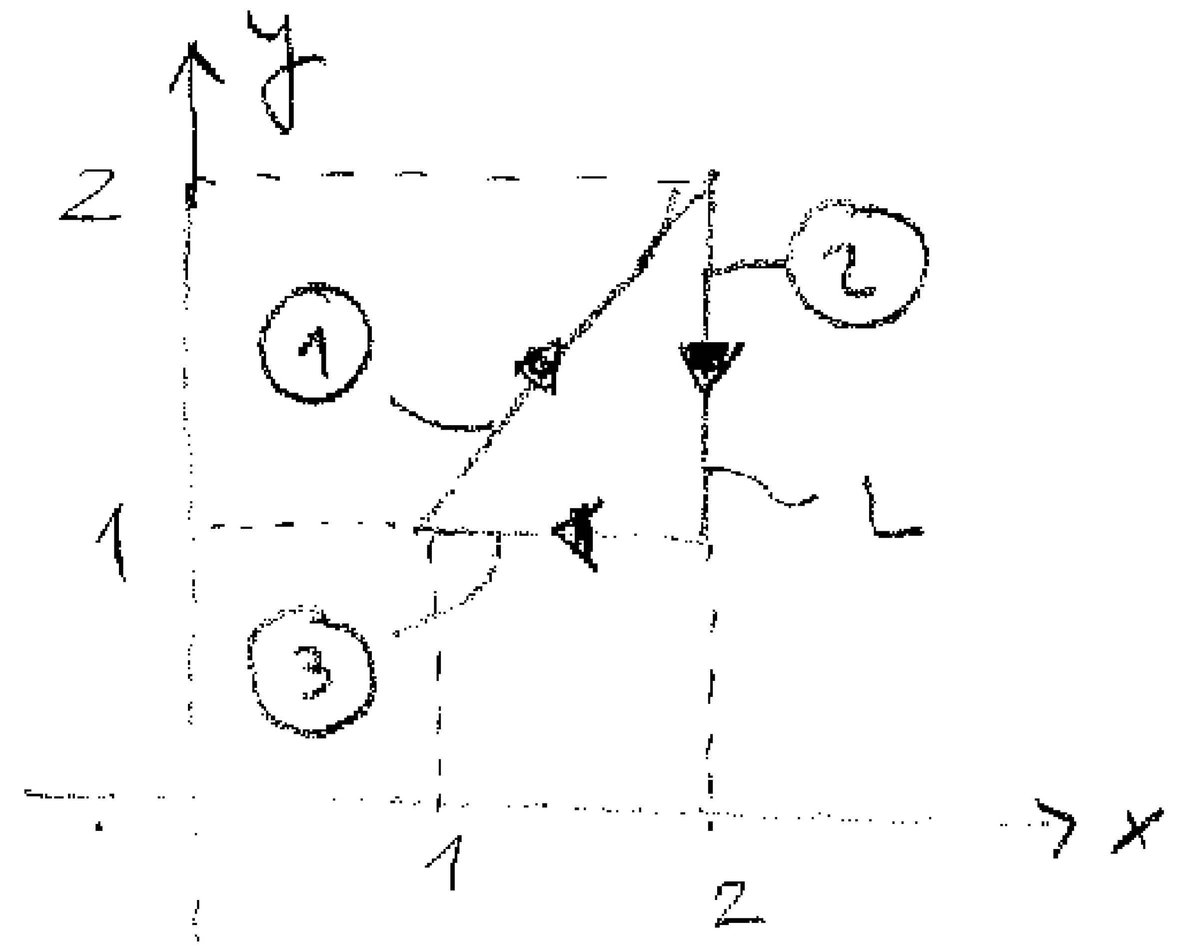
#1)



$$\vec{A} = 3x^2y^3\,\vec{a}_x - x^3y^2\,\vec{a}_y$$

$$\oint_L \vec{A}\cdot d\vec{l} = \left(\int_{①} + \int_{②} + \int_{③}\right)\vec{A}\cdot d\vec{l}$$

segment-①; $y = x$; $dy = dx$, $d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y$

$$\int_{①}\vec{A}\cdot d\vec{l} = \int_{①}(3x^2y^3\,dx - x^3y^2\,dy) = \int (3x^2x^3 - x^3x^2)\,dx$$

$$= \int_{x=1}^{2} 2x^5\,dx = 2\,\frac{x^6}{6}\Big|_1^2 = \frac{1}{3}(2^6 - 1^6) = \frac{63}{3}$$

segment-②; $x = 2$, $d\vec{l} = dy\,\vec{a}_y$

$$\int_{②}\vec{A}\cdot d\vec{l} = -\int_{②} x^3y^2\,dy = -8\int_{y=2}^{1} y^2\,dy = -8\,\frac{y^3}{3}\Big|_2^1$$

$$= -\frac{8}{3}(1-8) = \frac{56}{3}$$

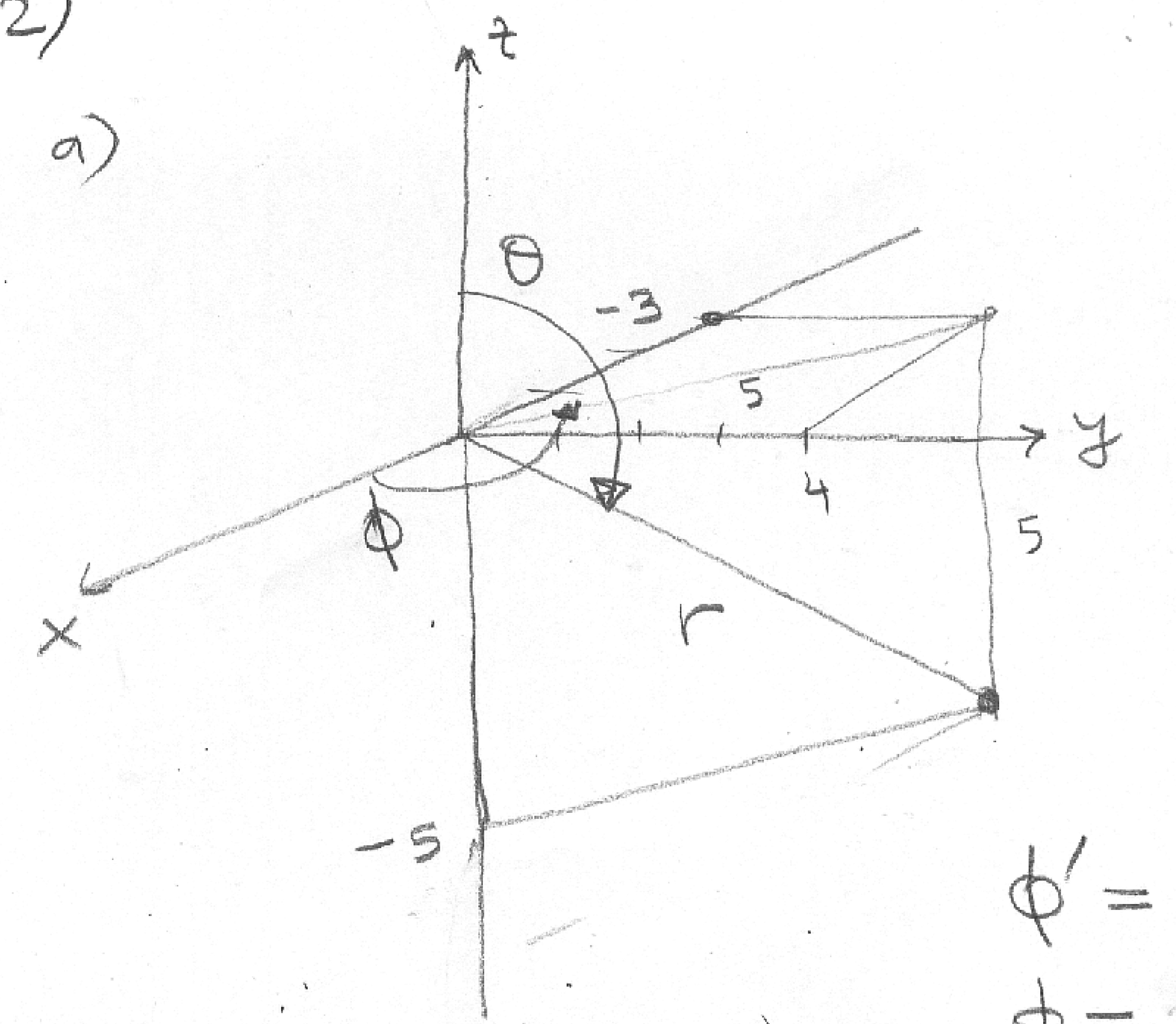
segment-③; $y = 1$, $d\vec{l} = dx\,\vec{a}_x$

$$\int_{③}\vec{A}\cdot d\vec{l} = \int_{③} 3x^2y^3\,dx = 3\int_{x=2}^{1} x^2\,dx = \cancel{3}\,\frac{x^3}{\cancel{3}}\Big|_2^1 = (1-8) = -7$$

$$\oint_L \vec{A}\cdot d\vec{l} = \frac{63}{3} + \frac{56}{3} - \frac{21}{3} = \frac{98}{3} \quad ※$$

#2)

a)



$r = \sqrt{9+16+25} = \sqrt{50}$

$r = 5\sqrt{2}$

$|\vec{E}| = \frac{25}{50} = \frac{1}{2} = 0{,}5$

$\phi' = \tan^{-1}\left(\frac{3}{4}\right) = 36{,}87^\circ$

$\phi = 90^\circ + 36{,}87 = 126.8^\circ$

$\theta' = \tan^{-1}\left(\frac{5}{5}\right) = 45^\circ$

$\theta = 90^\circ + 45^\circ = 135^\circ$

$E_x = \sin\theta \cos\phi \frac{25}{r^2} = \sin(135^\circ)\cos(126{,}8^\circ)\frac{25}{50}$
$= -0.21178$

b)

$E_y = \sin\theta \sin\phi \frac{25}{r^2} = \frac{1}{2}\sin(135^\circ)\sin(126.8^\circ) = 0.28310$

$E_z = \cos\theta \frac{25}{r^2} = \frac{1}{2}\cos(135^\circ) = -0{,}35355$

$\vec{E}\cdot\vec{B} = (-0{,}21178 \times 2 - 2 \times 0{,}28310 + 1 \times (-0{,}35355))$
$= [(0{,}21178)^2 + (0{,}28310)^2 + (0{,}35355)^2]^{1/2} \cdot [4+4+1]^{1/2}$ $\cos\alpha$

$\cos\alpha = \frac{-1.34331}{(0.5)\times(3)} = -0{,}89554$

$\alpha = \cos(-0{,}89554)$

$\boxed{\alpha = 153{,}57^\circ}$

#3) $\vec{H} = \rho z \cos\phi \, \vec{a}_\rho + e^{-z} \sin\frac{\phi}{2} \, \vec{a}_\phi + \rho^2 \vec{a}_z$

$P(1, \pi/3, 0)$
$\quad \rho \quad \phi \quad z$

a)
$$\begin{bmatrix} H_r \\ H_\theta \\ H_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} H_\rho \\ H_\phi \\ H_z \end{bmatrix}$$

At point $P(1, \pi/3, 0)$

$H_\rho = 0$

$H_\phi = \sin\frac{\pi}{6} = 0{,}5$

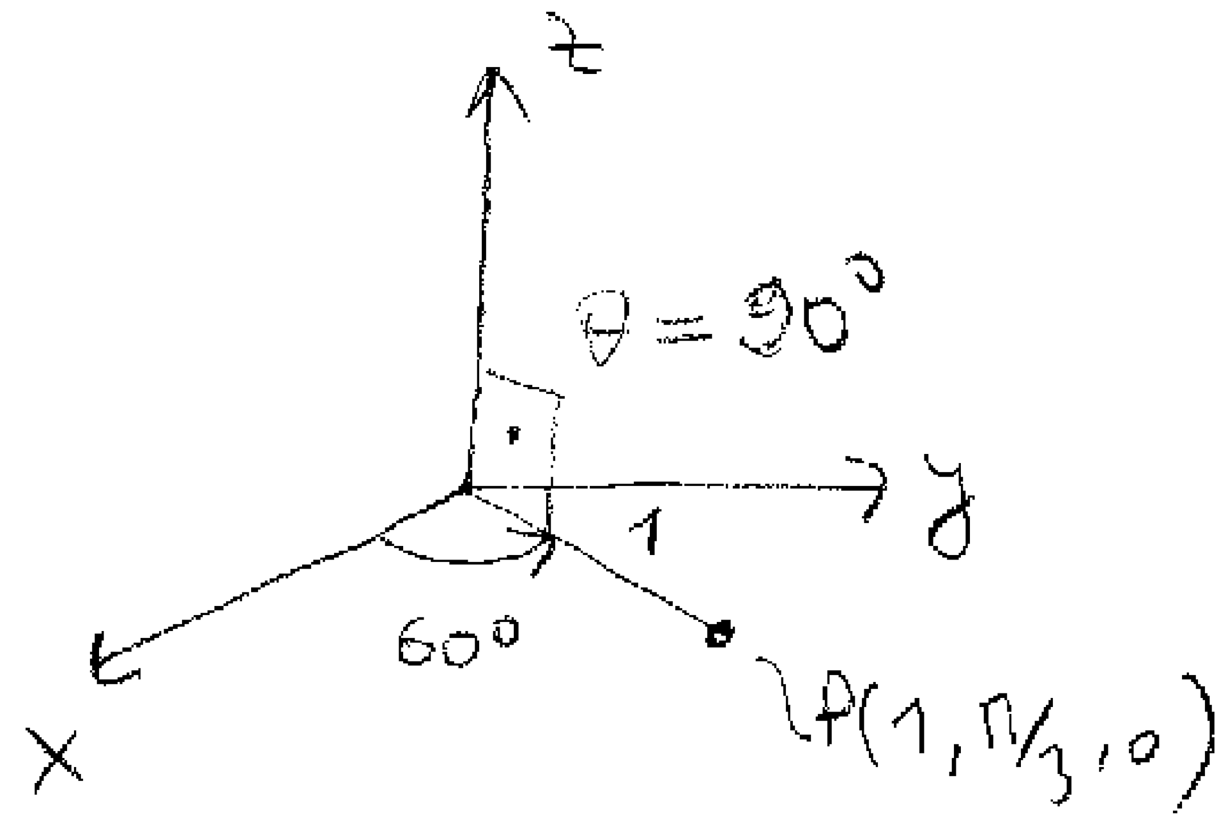
$H_z = 1$

$H_r = \sin 90^\circ \, H_\rho + \cos 90^\circ \, H_z = 0$

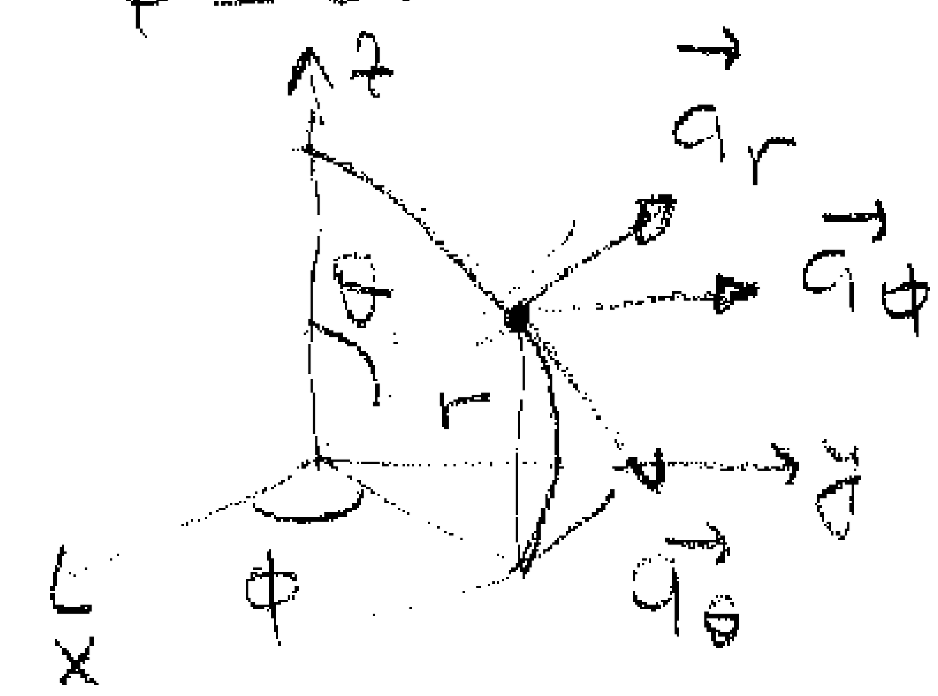
$H_\theta = \cos 90^\circ \, H_\rho - \sin 90^\circ \, H_z = -1$

$H_\phi = H_\phi = 0{,}5$

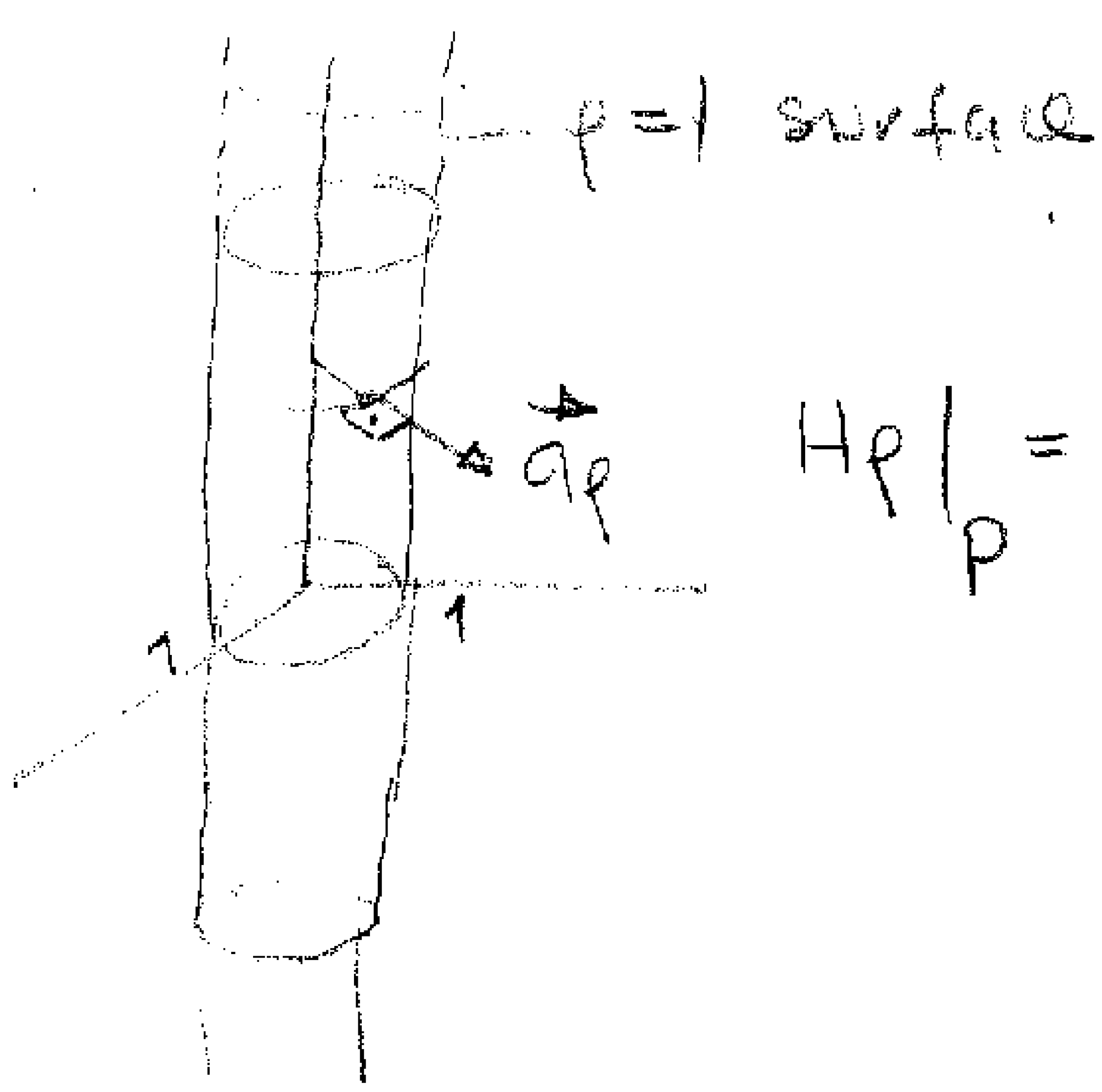
$(-\vec{a}_\theta + 0{,}5 \vec{a}_\phi) \times \vec{a}_\theta = -0{,}5 \vec{a}_r$
$= -0{,}5 \vec{a}_\rho$



$\theta = 90^\circ$
$\rho = r = 1$
$\phi = 60^\circ$



b)



$H_\rho \big|_P = 0 \; ; \; \vec{H}_\rho = 0$

c) $z=0$ plane $\rightarrow$ xy-plane ; scalar component of $\vec{H}$ that is tangential to $z=0$, is

$\vec{H}_{tan} = H_\rho \vec{a}_\rho + H_\phi \vec{a}_\phi$

$H_{tan} = [H_\rho^2 + H_\phi^2]^{1/2} = 0{,}5$