# ELECTROMAGNETICS I FIRST MIDTERM EXAM

**Dr. Salih FADIL** **March 22, 2012**

**#1)** Given that $\vec{Q} = (2x - y)\vec{a}_x + (4y + z)\vec{a}_y + (4x - 2z)\vec{a}_z$

**a)** determine a unit vector in the direction of $\vec{Q}$ at $P(1,2,1)$

**b)** find the component of $\vec{Q}$ in the direction of *PT* where *T* is point $(5,3,-4)$

**c)** where is $\vec{Q}$ the same as the unit vector of $\vec{a}_x + 11\vec{a}_y + 10\vec{a}_z$

**#2)** Given a vector field $\vec{D} = r\sin\phi\,\vec{a}_r - \frac{1}{r}\sin\theta\cos\phi\,\vec{a}_\theta + r^2\vec{a}_\phi$, determine:

**a)** $\vec{D}$, in spherical coordinates, at $P(4.330, -2.5, -8.660)$

**b)** the spherical vector component of $\vec{D}$ tangential to the surface of $r = 10$ at *P*

**c)** a unit vector in spherical coordinates at *P* normal to $\vec{D}$ and tangential to cone $\theta = 150^0$

**#3)** Given a vector field $\vec{A} = \rho^2\cos^2\phi\,\vec{a}_\rho + z\sin\phi\,\vec{a}_\phi$, show that the divergence theorem is satisfied over the closed surface described by $0 \le z \le 1$, $\rho = 4$ by this field.

**GOOD LUCK...☺**

(1)

# ELECTROMAGNETICS-I FIRST MIDTERM EXAM

## SOLUTION MANUAL

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#1)

a) $\vec{a}_\phi = \dfrac{\vec{Q}(1,2,1)}{|\vec{Q}(1,2,1)|} = \dfrac{0\,\vec{a}_x + 9\,\vec{a}_y + 2\,\vec{a}_z}{\sqrt{81+4}} = \underline{\underline{0{,}976\,\vec{a}_y + 0{,}217\,\vec{a}_z}}$

b) $\vec{r}_{PT} = \vec{r}_T - \vec{r}_P = (5,3,-4) - (1,2,1) = (4,1,-5)$

$= 4\vec{a}_x + \vec{a}_y - 5\vec{a}_z$

$\vec{Q}\cdot\vec{r}_{PT} = (Q \cdot r_{PT} \cos\theta) \Rightarrow \vec{Q}_{r_{PT}} = (\vec{Q}\cdot\vec{a}_{r_{PT}})\cdot\vec{a}_{r_{PT}}$

$\vec{a}_{r_{PT}} = \dfrac{(4,1,-5)}{(16+1+25)^{1/2}} = \dfrac{1}{\sqrt{42}}(4,+1,-5)$

$\vec{Q}_{r_{PT}} = 9 \times \dfrac{1}{\sqrt{42}} + \dfrac{-10}{\sqrt{42}} = \dfrac{-1}{\sqrt{42}}$ ; $\vec{Q}_{r_{PT}} = \dfrac{-1}{42}(4,1,-5)$

$\vec{Q}_{r_{PT}} = \underline{\underline{-0{,}0952\,\vec{a}_x - 0{,}0238\,\vec{a}_y + 0{,}119\,\vec{a}_z}}$ $\quad |\vec{Q}_{r_{PT}}| = 0{,}1542$

c) $\vec{A} = \vec{a}_x + 11\,\vec{a}_y + 10\,\vec{a}_z$

$\vec{a}_A = \dfrac{(1,11,10)}{(1+11^2+10^2)^{1/2}} = \dfrac{1}{\sqrt{222}}(1,11,10)$

$\dfrac{1}{\sqrt{222}} = 2x - y$

$2/ \dfrac{11}{\sqrt{222}} = 4y + z$
$\dfrac{10}{\sqrt{222}} = 4x - 2z$

$\dfrac{32}{\sqrt{222}} = 8y + 4x$

$\frac{1}{\sqrt{222}} = 2x - y \qquad \frac{40}{\sqrt{222}} = 20x \qquad x = \frac{2}{\sqrt{222}}$

$\frac{32}{\sqrt{222}} = 4x + 8y$

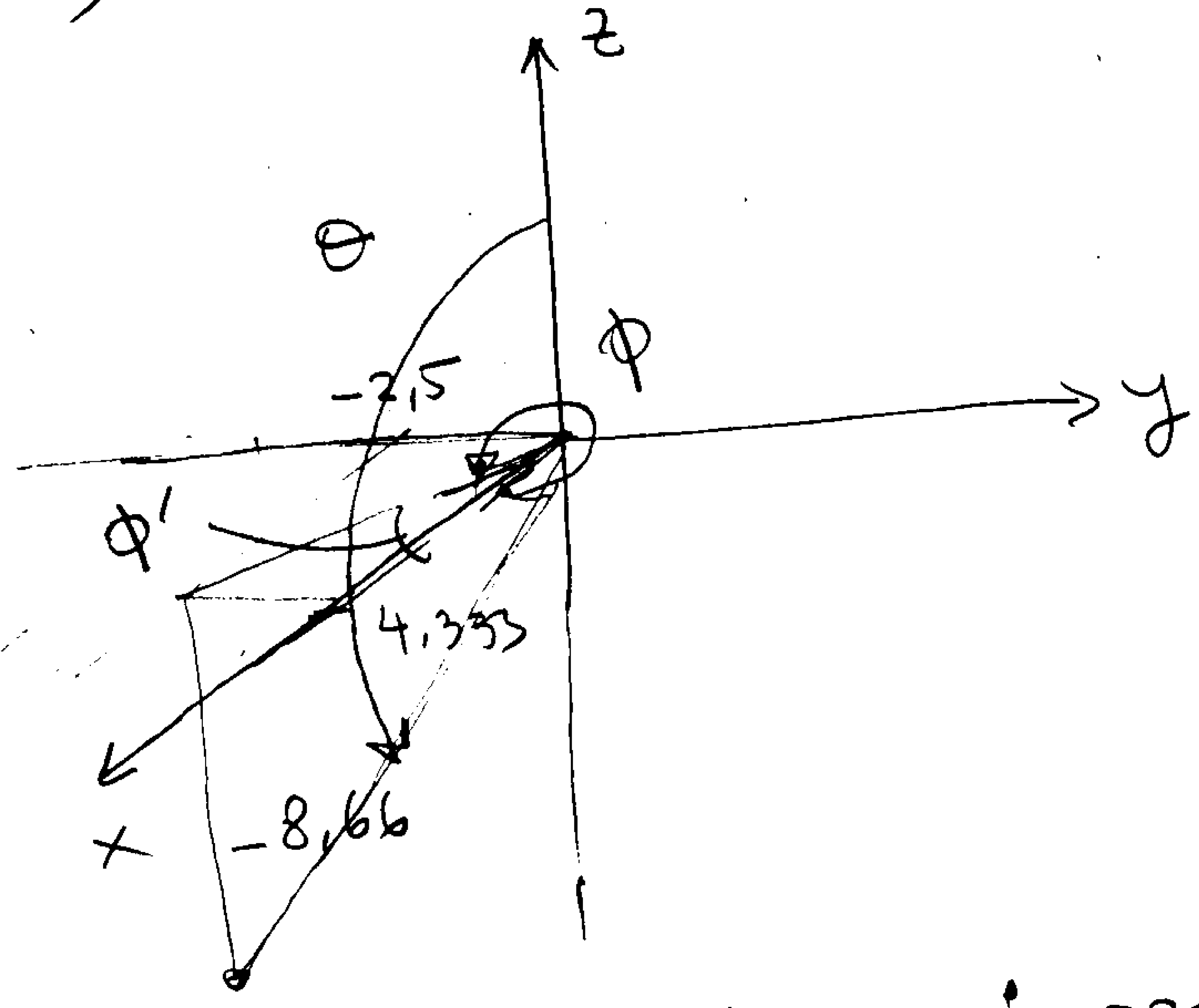
$y = 2x - \frac{1}{\sqrt{222}} = \frac{3}{\sqrt{222}}, \quad -z = 4y - \frac{11}{\sqrt{222}} = \frac{1}{\sqrt{222}}$

$\vec{Q}$ vector field becomes equal to $\vec{a}_A$ at point

$K\left(\frac{2}{\sqrt{222}}, \frac{3}{\sqrt{222}}, \frac{-1}{\sqrt{222}}\right) = (0{,}1342,\ 0{,}2013,\ -0{,}0671)$

#2) $\vec{D} = r\sin\phi\,\vec{a}_r - \frac{1}{r}\sin\theta\cos\phi\,\vec{a}_\theta + r^2\vec{a}_\phi$

a)




$r = \left[(4{,}33)^2 + (-2{,}5)^2 + (-8{,}66)^2\right]^{1/2}$

$r = 10$

$\phi = 360 - \phi' = 360 - \tan^{-1}\frac{2{,}5}{4{,}333}$

$\phi = 330^\circ$

$\theta = 90 + \tan^{-1}\frac{8{,}66}{\sqrt{2{,}5^2 + (4{,}33)^2}} = 150^\circ$

$P(r,\theta,\phi) = (10, 150^\circ, 330^\circ)$

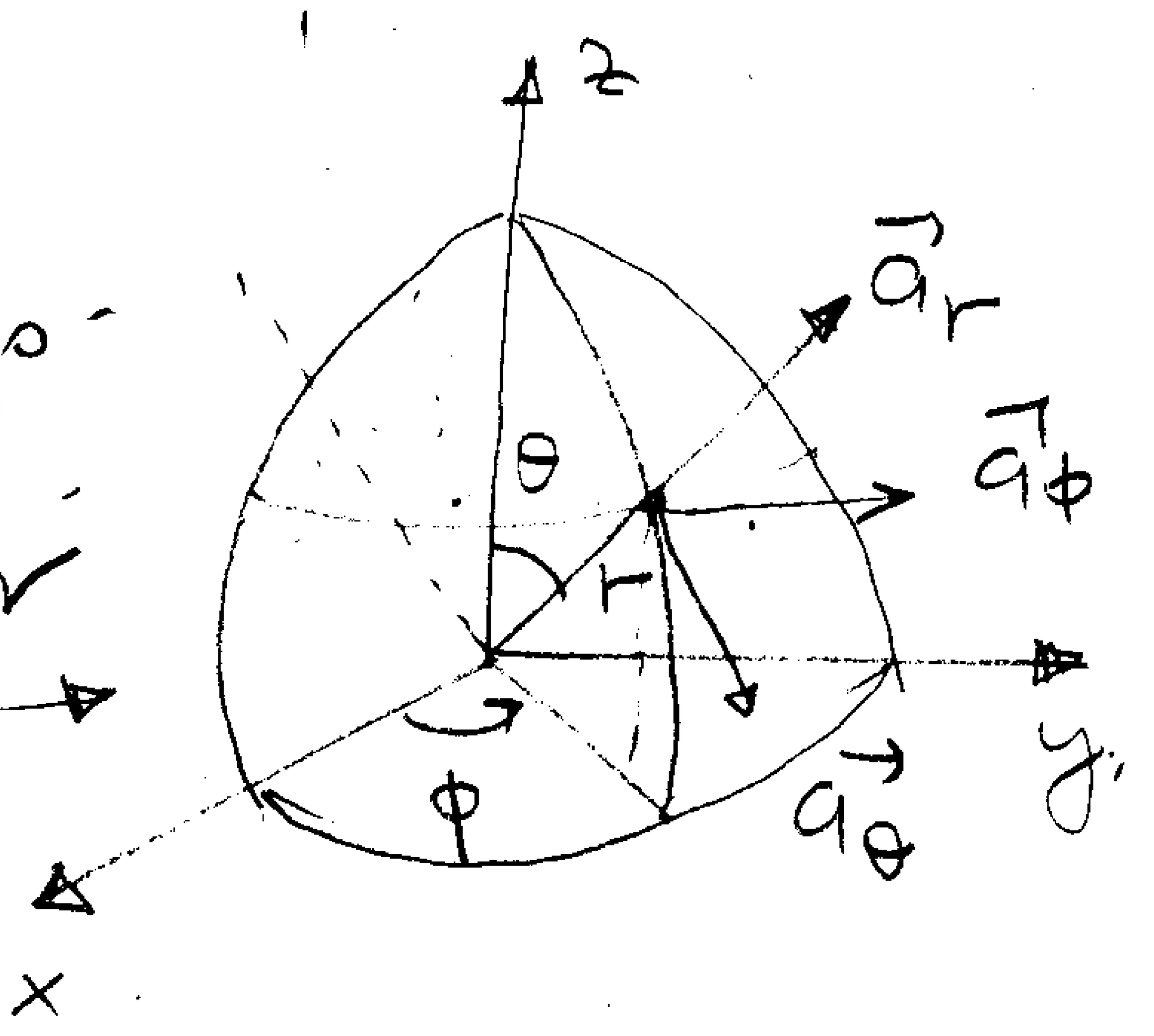
$\vec{D}\Big|_P = 10\sin(330^\circ)\,\vec{a}_r - \frac{1}{10}\sin(150^\circ)\cos(330^\circ)\,\vec{a}_\theta + (10)^2\,\vec{a}_\phi$

$= -5\,\vec{a}_r - 0{,}0433\,\vec{a}_\theta + 100\,\vec{a}_\phi$

b) $\vec{D} = \vec{D}_\perp + \vec{D}_{tan}$ at point P , $D_T = D_r \vec{a}_r$

$\vec{D}_{tan} = -0{,}0433\, \vec{a}_\theta + 100\, \vec{a}_\phi$

c) A vector at P normal to $\vec{D}$ and tangential to the cone $\theta = 150°$ is the same vector perpendicular to both $\vec{D}$ and $\vec{a}_\theta$ vector. Please see the figure.



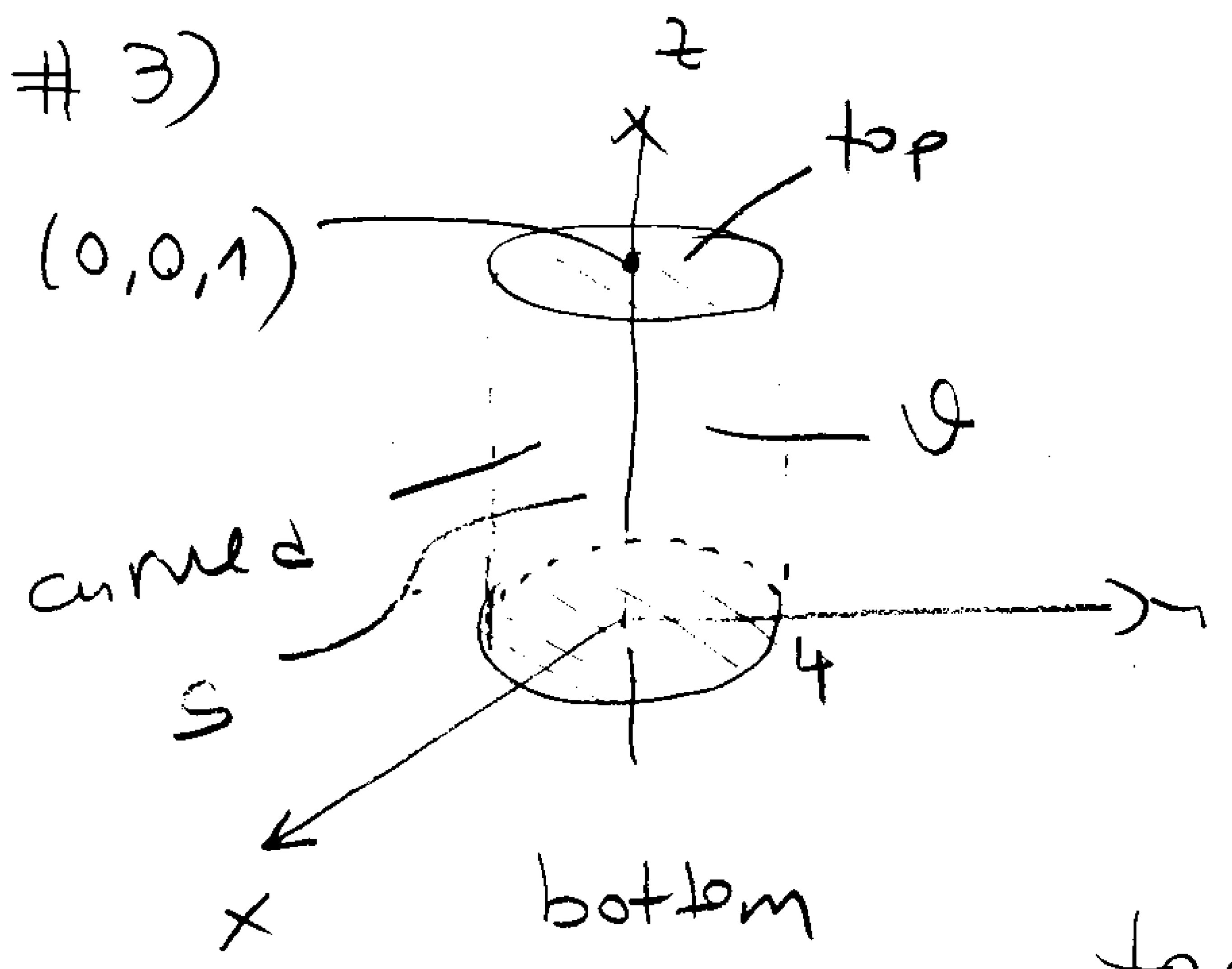

Therefore:

$$\vec{B} = \vec{D} \times \vec{a}_\theta = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\phi \\ -5 & -0{,}0433 & 100 \\ 0 & 1 & 0 \end{vmatrix} = \vec{a}_r (0-100) - \vec{a}_\theta (0-0) + \vec{a}_\phi (-5-0)$$

$$= -100\, \vec{a}_r - 5\, \vec{a}_\phi$$

$$\vec{a}_B = \frac{(-100, 0, -5)}{[(100)^2 + 25]^{1/2}} = -0{,}998\, \vec{a}_r - 0{,}05\, \vec{a}_\phi$$

# 3)




$$\oint_A \vec{A} \cdot \vec{ds} = \int_\vartheta (\nabla \cdot \vec{A}) \cdot dv$$

$$\oint_S A\, \vec{ds} = \left( \int_{top} + \int_{bottom} + \int_{curved} \right) \vec{A} \cdot \vec{ds}$$

top surface

$\vec{ds} = \rho\, d\phi\, d\rho\, \vec{a}_z$ ; $z = 1$

$$\int_{top} \vec{A}\cdot d\vec{S} = 0$$

Bottom surface

$$d\vec{S} = \rho\, d\phi\, d\rho\, (-\vec{a}_z) \longrightarrow \int_{bottom} \vec{A}\cdot d\vec{S} = 0$$

Curved surface

$$d\vec{S} = \rho\, d\phi\, dz\, \vec{a}_\rho \qquad \rho = 4$$

$$\int_{curved} \vec{A}\cdot d\vec{S} = \int_{curved} \rho^2 \cos^2\phi\, \rho\, d\phi\, dz = (4)^3 \int_{\phi=0}^{2\pi} \frac{1}{2}(1+\cos 2\phi)\, d\phi \int_{z=0}^{1} dz$$

$$= (4)^3\, \frac{1}{2}\left\{ \phi\Big|_0^{2\pi} + \cancel{\frac{1}{2}(\sin 2\phi)\Big|_0^{2\pi}} \right\} \cdot z\Big|_0^1 = (4)^3\, \frac{1}{\cancel{2}}\, \cancel{2}\pi \times 1$$

$$= 64\pi$$

$0$

$$\int_v (\nabla\cdot\vec{A})\, dv = ? \qquad \nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\underbrace{\rho\cdot\rho^2}_{\rho^3}\cos^2\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(z\sin\phi)$$

$$dv = \rho\, d\phi\, d\rho\, dz \qquad = \frac{1}{\rho}\, 3\rho^2 \cos^2\phi + \frac{1}{\rho}\, z\cos\phi$$

$$= \frac{1}{\rho}\left(3\rho^2\cos^2\phi + z\cos\phi\right)$$

$$\int_v (\nabla\cdot\vec{A})\, dv = \int_v \frac{1}{\cancel{\rho}}(3\rho^2\cos^2\phi + z\cos\phi)\,\cancel{\rho}\, d\phi\, d\rho\, dz$$

$$= 3\int_0^4 \rho^2\, d\rho \cdot \frac{1}{2}\int_0^{2\pi}(1+\cos 2\phi)\, d\phi \int_0^1 dz + \underbrace{\int_0^4 d\rho \int_0^{2\pi} \cancel{\cos\phi}\, d\phi \int_0^1 z\, dz}_{=0}$$

$0$ $0$ $0$

$$= \cancel{3}\,\frac{\rho^3}{\cancel{3}}\Big|_0^4 \cdot \frac{1}{2}\left[\phi\Big|_0^{2\pi} + \cancel{\frac{1}{2}\sin 2\phi\Big|_0^{2\pi}}\right] \cdot z\Big|_0^1$$

$$= (4)^3\, \frac{1}{\cancel{2}}\, \cancel{2}\pi = 64\pi$$

So, the divergence theorem is satisfied !