ELECTROMAGNETICS – I SECOND MIDTERM EXAM

Dr. Salih FADIL **May 10, 2012**

#1) A point charge 200 pC is located at (5m, 1m, -4m) while the y-axis carries 4 nC/m. If the planes z=4m and z=-3m carry charges 6 nC/m^2 and -8 nC/m^2 respectively, calculate $\bar{E}$ at (1m, 1m,1m).

#2) z-axis carries charge 10 nC/m, find the stored energy in the region defined by $0m \le \rho \le 5m, \quad -1m \le z \le 5m$.

#3) 100 nC point charge is located at the origin. Calculate the work done in moving 1 nC charge from point (1m, 1m, 1m) to point (10m, 20m, 30m). Who does the work?

GOOD LUCK...☺

ELECTROMAGNETICS SECOND MIDTERM

SOLUTION MANUAL

Dr. Salih FADIL

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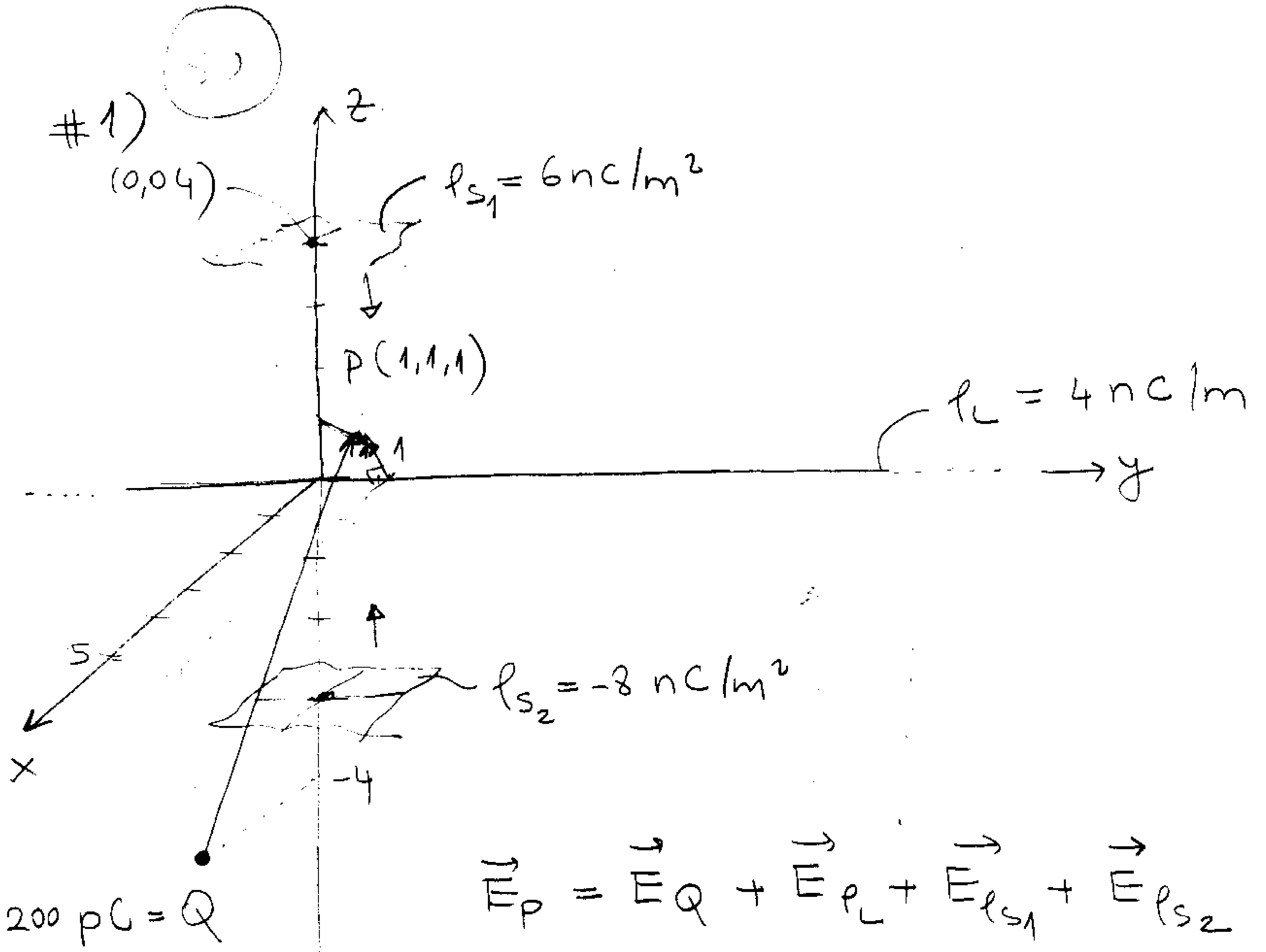
#1)



$$\vec{E}_P = \vec{E}_Q + \vec{E}_{\rho_L} + \vec{E}_{\rho_{s_1}} + \vec{E}_{\rho_{s_2}}$$

$$\vec{E}_Q = \frac{Q\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3\, 4\pi\varepsilon_0} = \frac{200\;10^{-12}}{\cancel{4\pi}\,\frac{10^{-9}}{\cancel{36\pi}_{\,9}}}\;\frac{\overbrace{\left((1,1,1)-(5,1,-4)\right)}^{(-4,\,0,\,5)}}{\underbrace{|(1,1,1)-(5,1,-4)|^3}_{|(-4,0,5)|^3}}$$

$$= \frac{1800\;10^{-3}}{(16+25)^{3/2}}\,(-4\,\vec{a}_x + 5\,\vec{a}_z)$$

$$\vec{E}_Q = -0{,}027425\,\vec{a}_x + 0{,}034282\,\vec{a}_z \ \text{V/m}$$

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$$\vec{E}_{\rho_L} = \frac{\rho_L}{2\pi\varepsilon_0} \frac{\vec{\rho}}{\rho^2}$$

$$\vec{\rho} = (1,1,1) - (0,1,0)$$
$$\vec{\rho} = (1,0,1) \qquad \rho = \sqrt{2}$$

$$\vec{E}_{\rho_L} = \frac{4\,10^{-9}}{\cancel{2\pi}\,\frac{10^{-9}}{\cancel{36\pi}_{18}}} \frac{\vec{a_x}+\vec{a_z}}{2} = \frac{18 \times 4}{\cancel{2}}(\vec{a_x}+\vec{a_z})$$

$$= \underline{36\vec{a_x} + 36\vec{a_z}} \text{ V/m.}$$

13

$$\vec{E}_{s_1} = \frac{\rho_{s1}}{2\varepsilon_0}(-\vec{a_z}) = \frac{\overset{3}{\cancel{6}}\,\cancel{10^{-9}}}{\cancel{2}\,\frac{\cancel{10^{-9}}}{36\pi}}(-\vec{a_z}) = -108\pi\vec{a_z}$$

$$= \underline{-339{,}3\,\vec{a_z}} \text{ V/m}$$

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$$\vec{E}_{s_2} = \frac{\rho_{s_2}}{2\varepsilon_0}(+\vec{a_z}) = \frac{\overset{4}{\cancel{-8}}\,\cancel{10^{-9}}}{\cancel{2}\,\frac{\cancel{10^{-9}}}{36\pi}}\vec{a_z} = -144\pi\vec{a_z}$$

$$= \underline{-452{,}39\,\vec{a_z}} \text{ V/m}$$

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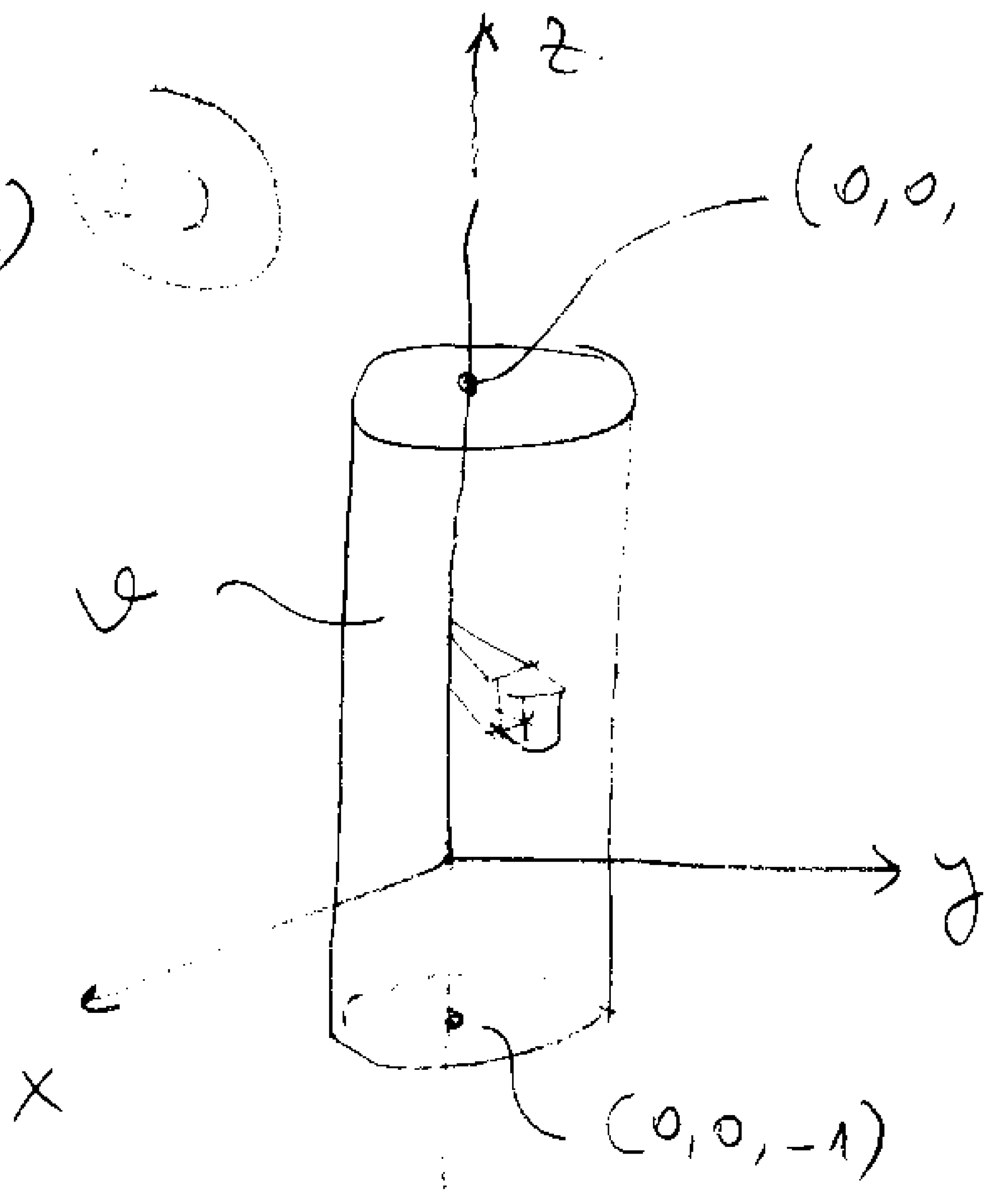
$$\boxed{\vec{E} = 35.9725\,\vec{a_x} - 755.69\,\vec{a_z} \quad \text{V/m}}$$

#2)



$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a_\rho}$$

$$W_E = \frac{1}{2}\varepsilon_0\int_v |\vec{E}|^2\,dv$$

$$= \frac{1}{2}\varepsilon_0\int_v \left(\frac{\rho_L}{2\pi\varepsilon_0}\right)^2 \frac{1}{\cancel{\rho^2}}\,\cancel{\rho}\,d\rho\,d\phi\,dz$$

$$= \frac{1}{2}\cancel{\varepsilon_0}\frac{\rho_L^2}{4\pi^2\cancel{\varepsilon_0}^2}\int_{\rho=0}^{5}\frac{d\rho}{\rho}\cdot\int_{-1}^{5}dz\int_0^{2\pi}d\phi$$

ε_0

$$= \frac{1}{2}\frac{\rho_L^2}{4\pi^2\varepsilon_0}\ln\rho\Big|_0^5\; z\Big|_{-1}^5\; 2\pi$$

Since $\ln 0 \to -\infty$

$W_E^1 \to$ undefined

#3) $\boxed{30}$

$$\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2}\vec{a_r} = \frac{100\ \cancel{10^{-9}}}{\cancel{4\pi}\ \frac{\cancel{10^{-9}}}{\cancel{36\pi}}\ r^2}\vec{a_r} = \frac{900}{r^2}\vec{a_r}\ \text{V/m}$$

$$A(1,1,1) \longrightarrow r_A = \sqrt{1+1+1} = \sqrt{3}\ \text{m}$$

$$B(10,20,30) \longrightarrow r_B = \sqrt{100+400+900} = \sqrt{1400} = 37{,}416\ \text{m}.$$

$$W = -10^{-9} \int_A^B \frac{900\,\vec{a_r}}{r^2}\cdot dr\,\vec{a_r} = +900\ 10^{-9}\int_{\sqrt{3}}^{37{,}416} -\frac{dr}{r^2}$$

$$= +900\ 10^{-9}\ \frac{1}{r}\Bigg|_{\sqrt{3}}^{37{,}416} = +900\ 10^{-9}\left(\frac{1}{37{,}416} - \frac{1}{\sqrt{3}}\right)$$

$$= -4{,}955\ 10^{-7}\ \text{J}.$$

$$= -0{,}495\ 10^{-6}\ \text{J}$$

$$= -0{,}495\ \mu\text{J} < 0$$ 25

field does the work 5