ELECTROMAGNETICS I FIRST MIDTERM EXAM

Dr. Salih FADIL **October 17, 2012**

#1) If $P_1(1,2,-3)$ and $P_2(-4,0,5)$, find:

a) a point P_3 such that points P_1, P_2 and P_3 on the same line and the distance between P_1 and P_3 is 20.

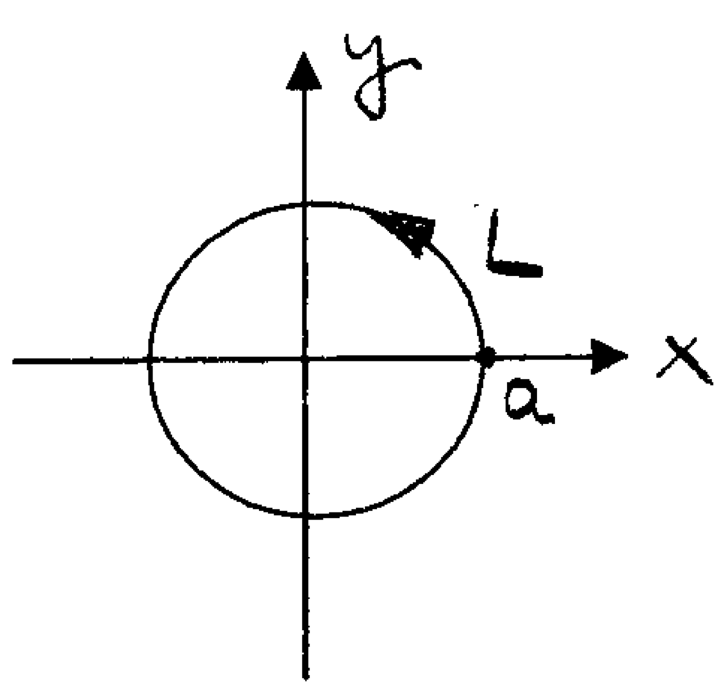
b) The shortest distance between the line and point $P_4(7,-1,2)$.

#2) Two uniform vector fields are given by $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z$ and $\vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z$. Calculate:

a) the vector component of $\vec{E}$ at $P(5, \frac{\pi}{2}, 3)$ parallel to the line $x = 2, z = 3$

b) the angle $\vec{E}$ makes with the surface $z = 3$ at P.

#3) Consider the line integral $\oint_L \frac{-y\vec{a}_x + x\vec{a}_y}{\sqrt{x^2+y^2}} . \overrightarrow{d\ell}$ where L is a circle of radius a in xy-plane, with center at the origin, traced out counterclockwise as shown in the figure. Express the integrant in cylindrical coordinate system first and then calculate the line integral



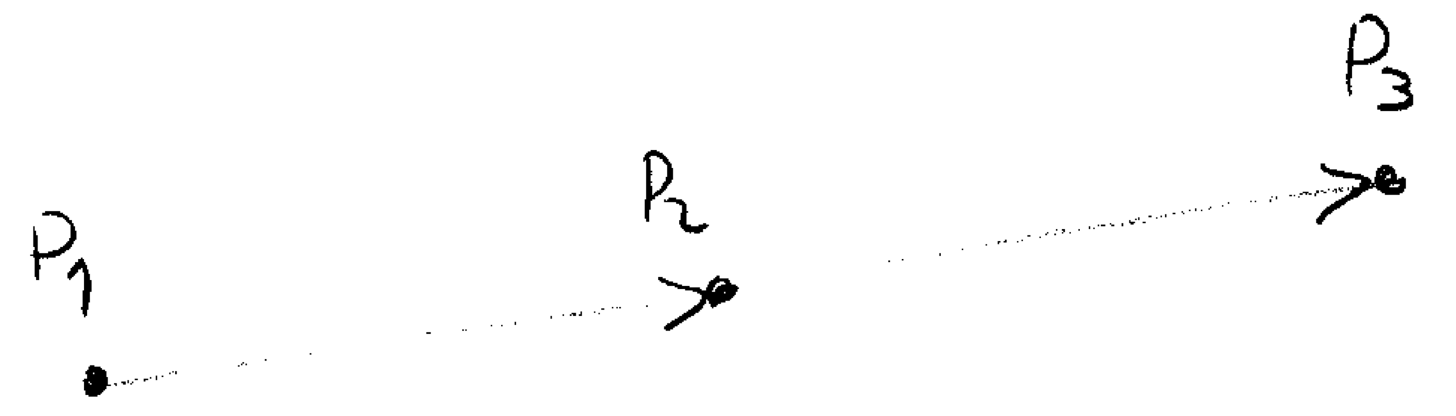
GOOD LUCK...☺

ELECTROMAGNETICS-I FIRST MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

October 17, 2012

#1) $P_1(1,2,-3)$, $P_2(-4,0,5)$ $P_3(x,y,z)$



$\vec{r}_{P_1P_2} = (-4,0,5)-(1,2,-3)$
$= (-5,-2,8)$

$\vec{r}_{P_1P_3} = \left((x-1),\ (y-2),\ (z+3)\right)$

if P_1, P_2 and P_3 are on the same line $\Rightarrow \vec{r}_{P_1P_2} \parallel \vec{r}_{P_1P_3}$

$x-1 = -5\alpha$
$y-2 = -2\alpha$
$z+3 = 8\alpha$

$|\vec{r}_{P_1P_3}| = 20$ $\left(25\alpha^2+4\alpha^2+64\alpha^2\right)^{1/2} = 20$

$\sqrt{93\alpha^2} = 20$ $\alpha = \dfrac{20}{\sqrt{93}}$

$\alpha = 2.0739$

$x = 1-5\times\dfrac{20}{\sqrt{93}} = -9.3695$

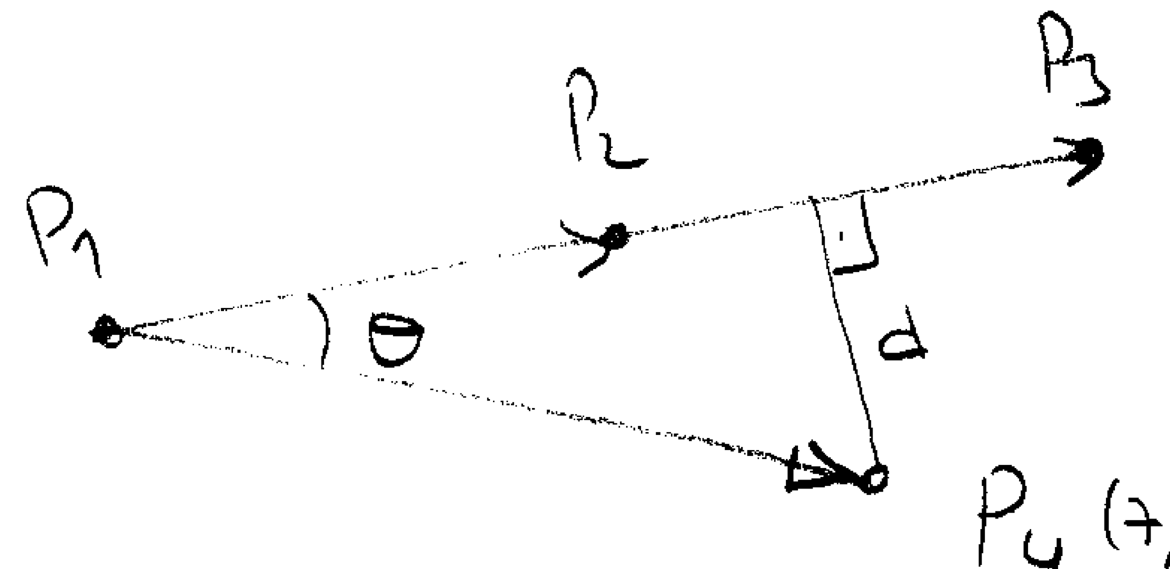
$y = 2-2\alpha = 2-2\times\dfrac{20}{\sqrt{93}} = -2.1478$

$z = 8\alpha-3 = 8\times\dfrac{20}{\sqrt{93}}-3 = 13.5912$

$P_3(-9.3695,\ -2.1478,\ 13.5912)$

Note that another P_3 point can be found by taking $\alpha = -\dfrac{20}{\sqrt{93}}$ since $\alpha>0$ or $\alpha<0$

b)



$|\vec{r}_{P_1P_2}\times\vec{r}_{P_1P_4}| = r_{P_1P_2}\,\underbrace{r_{P_1P_4}\sin\theta}_{d}$

d = it is the shortest distance between the line and P_4

$$\vec{r}_{P_1P_2} \times \vec{r}_{P_1P_4} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -5 & -2 & 8 \\ 6 & -3 & 5 \end{vmatrix}$$

$$\vec{r}_{P_1P_4} = (7,-1,2)-(1,2,-3) = (6,-3,5)$$

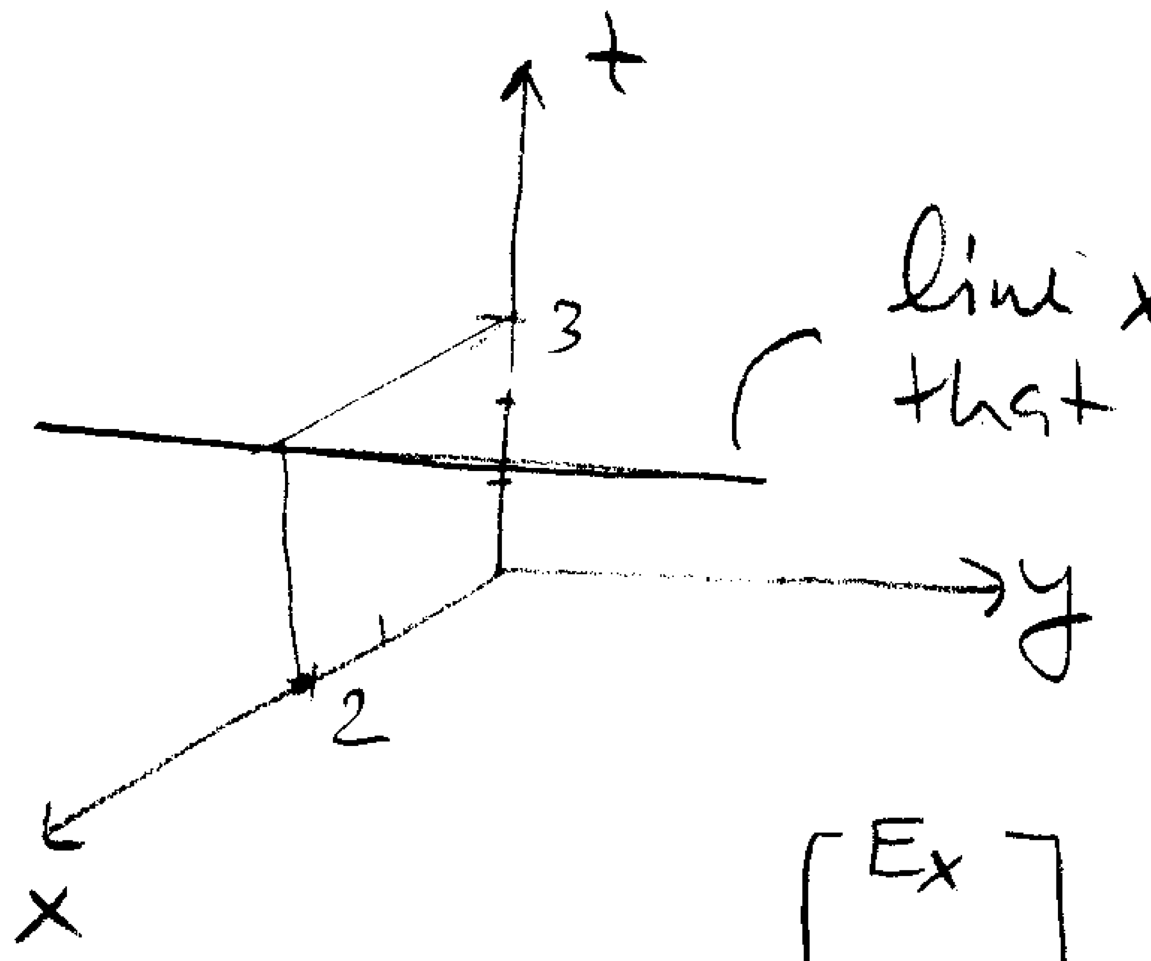
$$= \vec{a}_x(-10+24) - \vec{a}_y(-25-48) + \vec{a}_z(15+12)$$

$$= 14\,\vec{a}_x - 73\,\vec{a}_y + 27\,\vec{a}_z$$

$$d = \frac{(14^2+73^2+27^2)^{1/2}}{(5^2+2^2+8^2)^{1/2}} = \frac{\sqrt{6254}}{\sqrt{93}} = 8.2 \text{ unit}$$

#2)

a) $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z$



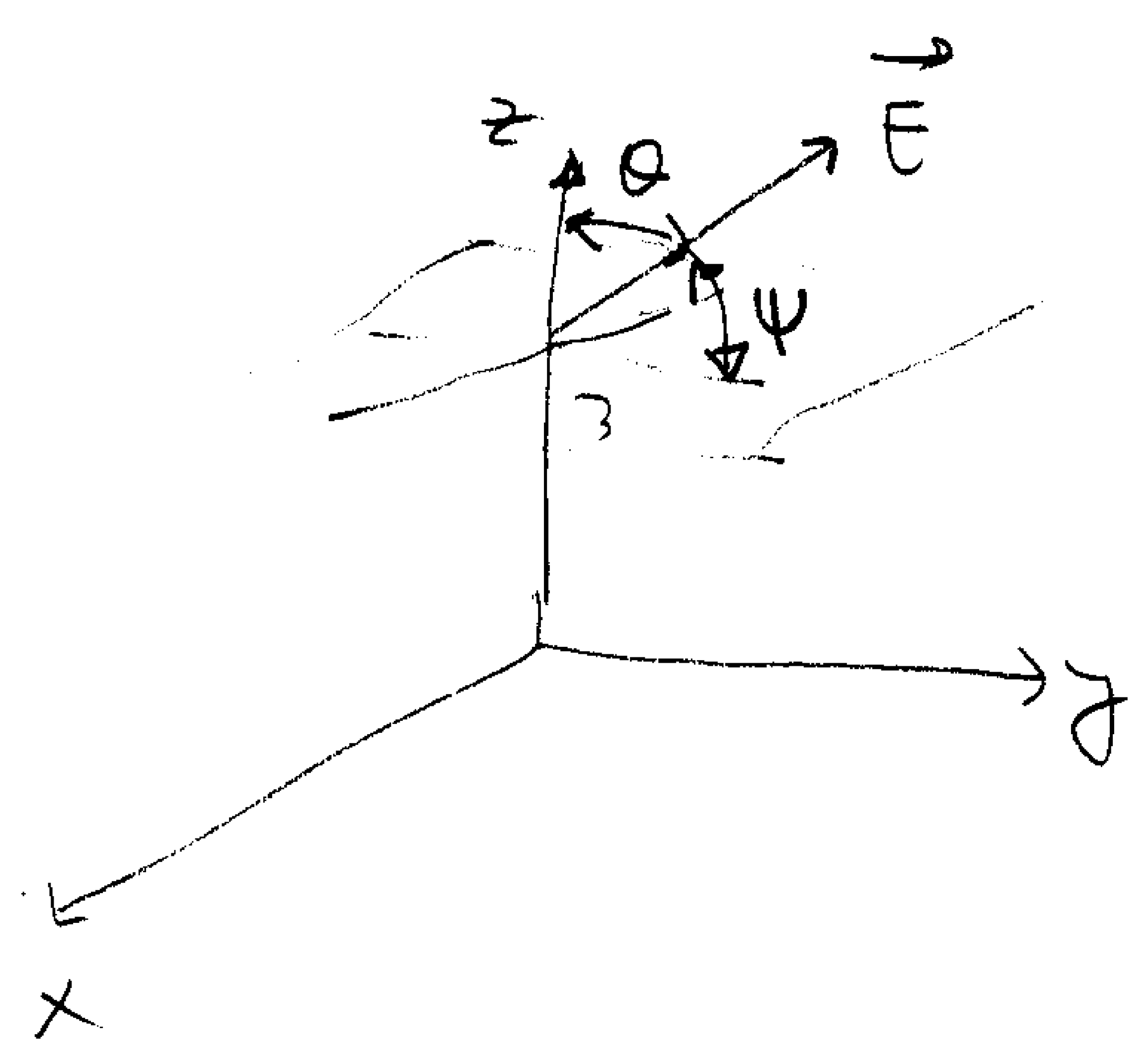
line $x=2, z=3$ that is parallel to y-axis

the component of $\vec{E}$ that is parallel to the line is E_y, so

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \qquad \phi = \pi/2$$

$E_y = \sin(\pi/2)(-5) = -5$ so $\vec{E}_y = -5\vec{a}_y$

b)



$$\theta + \psi = \pi/2$$

$$\vec{E} \cdot \vec{a}_z = |\vec{E}| \cdot 1 \cos\theta$$

$E_x = -1 \cdot 10 = -10$

$E_y = -5 \times 1 = -5$

$E_z = 3$

$$\vec{E} = -10\,\vec{a}_x - 5\,\vec{a}_y + 3\,\vec{a}_z$$

$$3 = (100 + 25 + 9)^{1/2}\cos\theta \qquad \theta = \cos^{-1}\left(\frac{3}{\sqrt{134}}\right) = 74.98^\circ$$

$$\psi = 90 - 74.98^\circ = \underline{\underline{15.02^\circ}}$$

#3) $$\vec{F} = \frac{-y}{\sqrt{x^2+y^2}}\,\vec{a}_x + \frac{x}{\sqrt{x^2+y^2}}\,\vec{a}_y$$

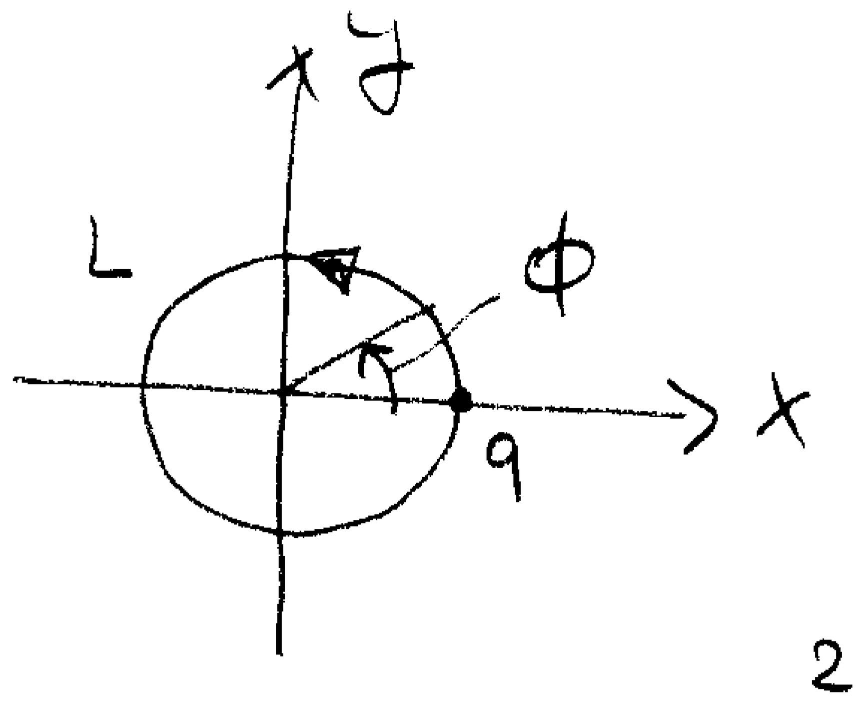
$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ 0 \end{bmatrix} \qquad \begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \end{aligned}$$

$$F_\rho = \cos\phi\, F_x + \sin\phi\, F_y = \cos\phi\,\frac{-\cancel{\rho}\sin\phi}{\cancel{\rho}} + \sin\phi\,\frac{\cancel{\rho}\cos\phi}{\cancel{\rho}} = 0$$

$$F_\phi = -\sin\phi\, F_x + \cos\phi\, F_y = -\sin\phi\,\frac{-\cancel{\rho}\sin\phi}{\cancel{\rho}} + \cos\,\frac{\cancel{\rho}\cos\phi}{\cancel{\rho}}$$

$$= \sin^2\phi + \cos^2\phi = 1$$

$$\vec{F} = \vec{a}_\phi$$



$$\vec{dl} = \rho\, d\phi = a\, d\phi\, \vec{a}_\phi$$

$$\int_L \vec{F}\cdot\vec{dl} = a\int_L d\phi = a\int_0^{2\pi} d\phi = a\,\phi\Big|_0^{2\pi} = 2\pi a$$