ELECTROMAGNETICS I SECOND MIDTERM EXAM

Dr. Salih FADIL **August 05, 2011**

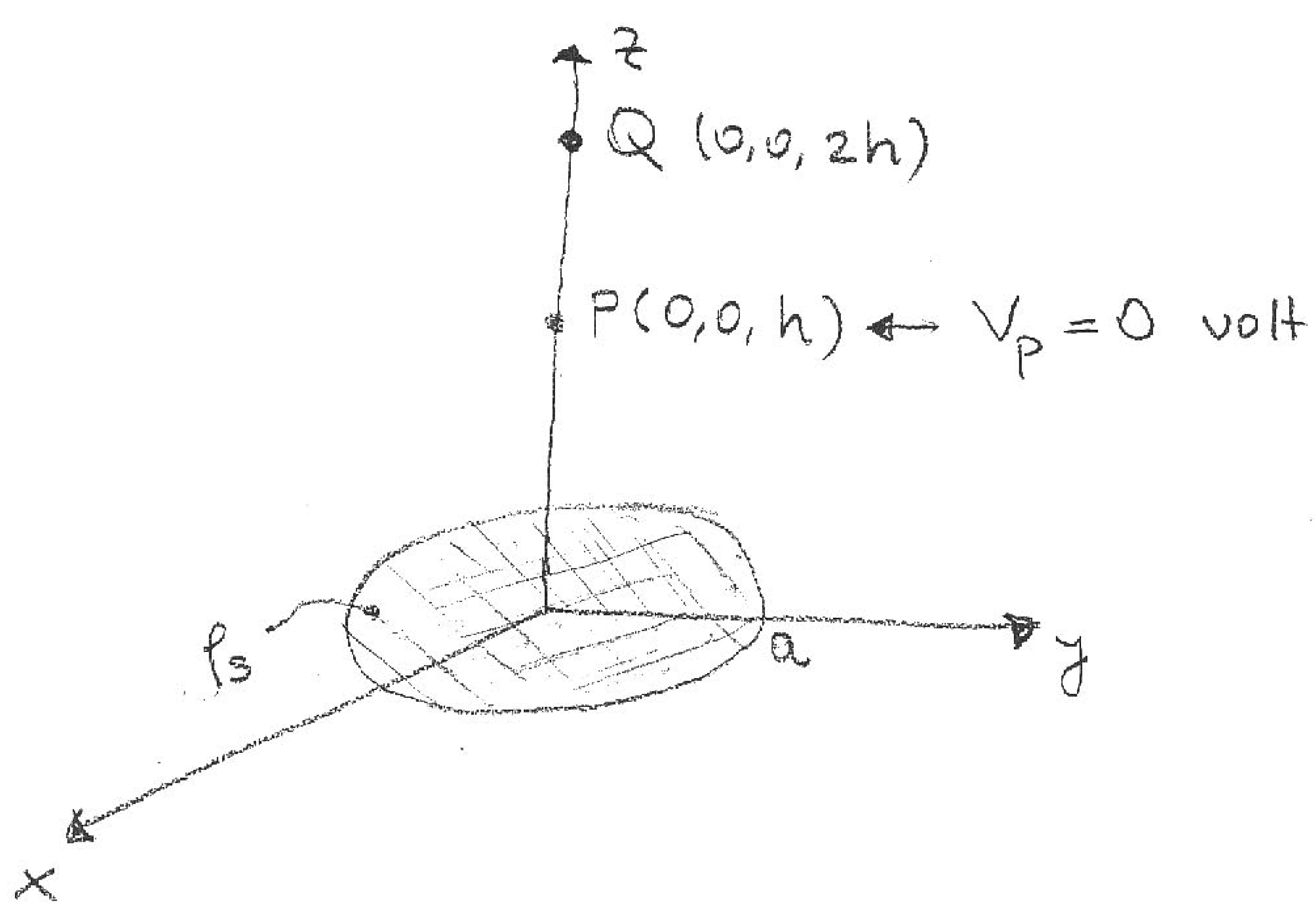
#1) Given that $\vec{E} = (3x^2 + y)\vec{a}_x + x\vec{a}_y$ kV/m, find the work done in moving a -2 μC charge from $(0, 5, 0)$ to $(2, -1, 0)$ by taking the path

a) $(0,5,0) \rightarrow (2,5,0) \rightarrow (2,-1,0)$

b) $y = 5 - 3x$

#2) If $x = 3$, $y = -4$ respectively carry charges $15\, nc/m^2$ and $20\, nc/m^2$. If the line $x = 0$, $z = -2$ carries a charge $10\pi\ nC/m$, Calculate $\bar{E}$ at (1, 1, -1) due to three charge distributions.

#3) If the potential (absolute) at point P is zero in the following figure, find ρ_s in terms of Q.



ELECTROMAGNETICS I SECOND MIDTERM EXAM.

SOLUTION MANUAL

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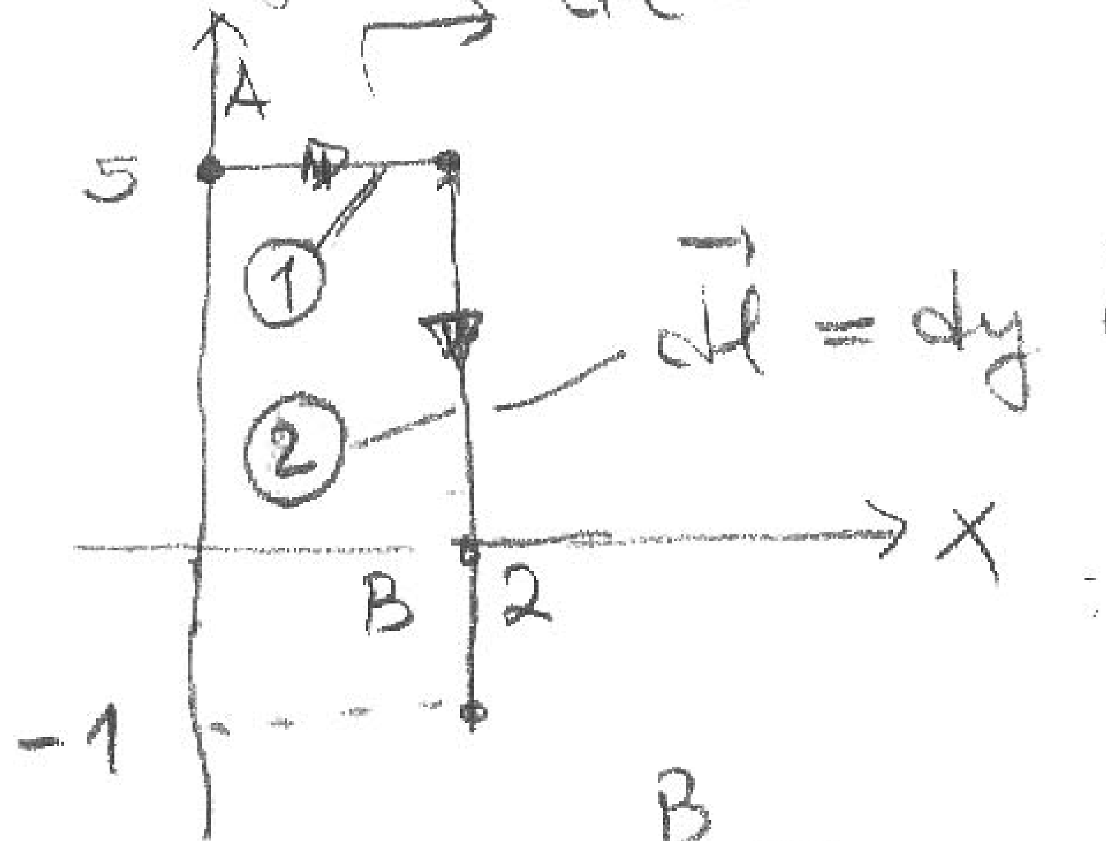
#1) $\vec{E} = (3x^2+y)\,\vec{a}_x + x\,\vec{a}_y$ kV/m. , $Q = -2\,\mu C$

a)



$$W = -Q\int_A^B \vec{E}\cdot\vec{dl} = +2\,10^{-6}\left\{ \underbrace{\int (3x^2+y)\,dx}_{1} + \underbrace{\int x\,dy}_{2} \right\} 10^3$$

$$= 2\,10^{-6}\left[\int_0^2 (3x^2+5)\,dx + \int_5^{-1} 2\,dy\right] 10^3$$

$$= 2\,10^{-3}\left[\cancel{3}\frac{x^3}{\cancel{3}}\Big|_0^2 + 5x\Big|_0^2 + 2y\Big|_5^{-1}\right\} = 2\,10^{-3}\left[(8-0)+(10-0) + (-2-10)\right]$$

$$= 2\,10^{-3}(18-12) = 12\ mJ > 0$$

external agent does the work

b) $\vec{dl} = dx\,\vec{a}_x + dy\,\vec{a}_y \qquad dy = -3\,dx$

$$W = +2\,10^{-6}\left\{\int_A^B \left[(3x^2+y)\,dx + x\,dy\right] 10^3\right.$$

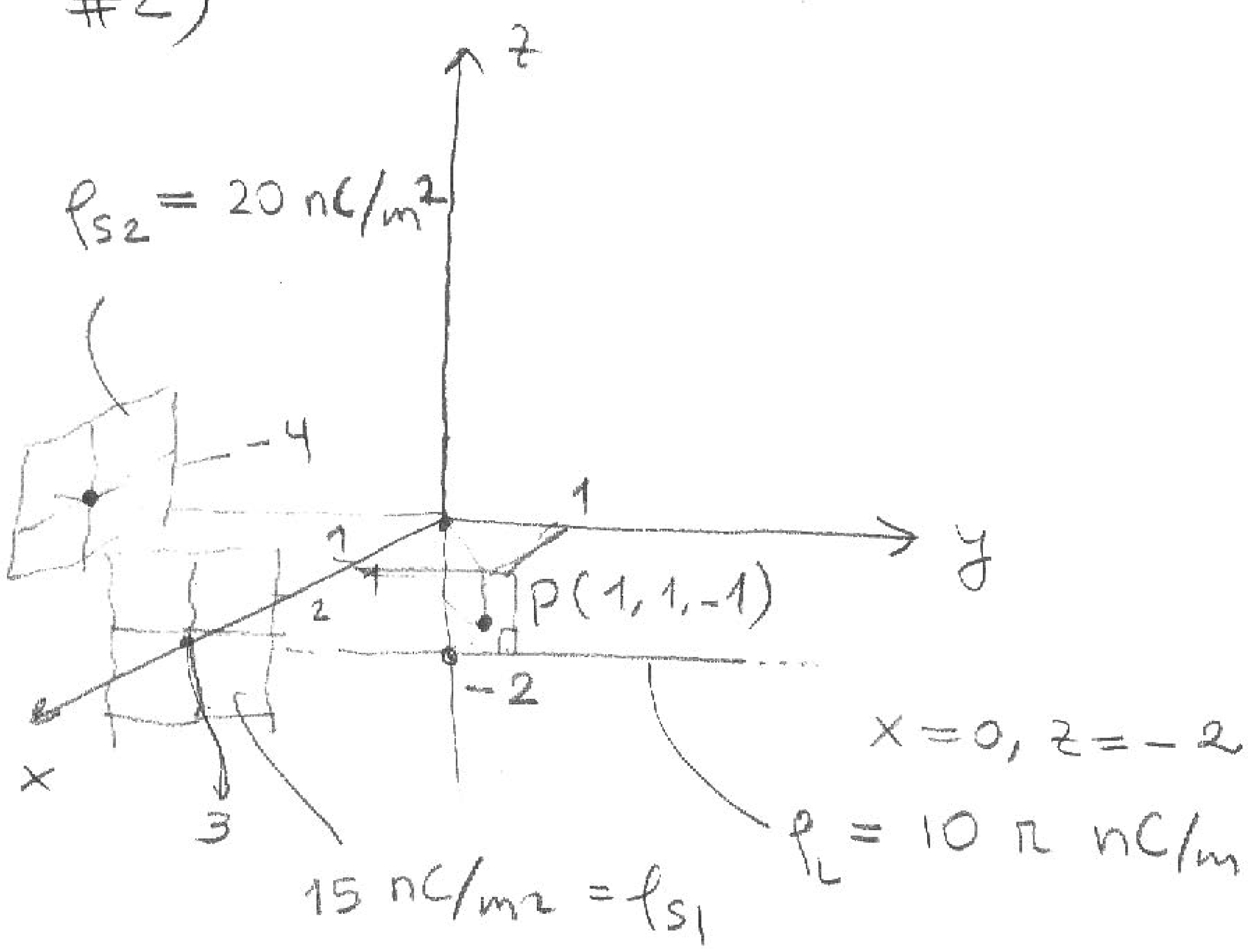
$$= 2\,10^{-3}\left\{\int \left[3x^2+5-3x + x(-3)\right] dx\right\}$$

$$= 2\,10^{-3}\left\{\int_0^2 (3x^2-6x+5)\,dx\right\} = 2\,10^{-3}\left\{\left(\cancel{3}\frac{x^3}{\cancel{3}} - 6\frac{x^2}{2} + 5x\right)\Big|_0^2\right.$$

$$= 2\,10^{-3}\left\{8-3\times4+10\right\} = 12\,10^{-3}\ J$$

$$= 12\ mJ$$

#2)



$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n \qquad \vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a}_\rho$$

$$\vec{E}_{s1} = \frac{15\cancel{10^{-9}}}{2\frac{\cancel{10^{-9}}}{\cancel{36}\pi}}(-\vec{a}_x) = -15 \times 18\pi\,\vec{a}_x = -270\pi\,\vec{a}_x \ \ V/m$$

18

$$\vec{E}_{s2} = \frac{20\cancel{10^{-9}}}{\cancel{2}\frac{\cancel{10^{-9}}}{\cancel{36}\pi}}(\vec{a}_y) = 360\pi\,\vec{a}_y \ \ V/m$$

18

$$\vec{\rho} = (1,1,-1) - (0,1,-2) = (1,0,1) = \vec{a}_x + \vec{a}_z$$

$$\rho = \sqrt{2} \qquad \vec{a}_\rho = \frac{1}{\sqrt{2}}(\vec{a}_x + \vec{a}_z)$$

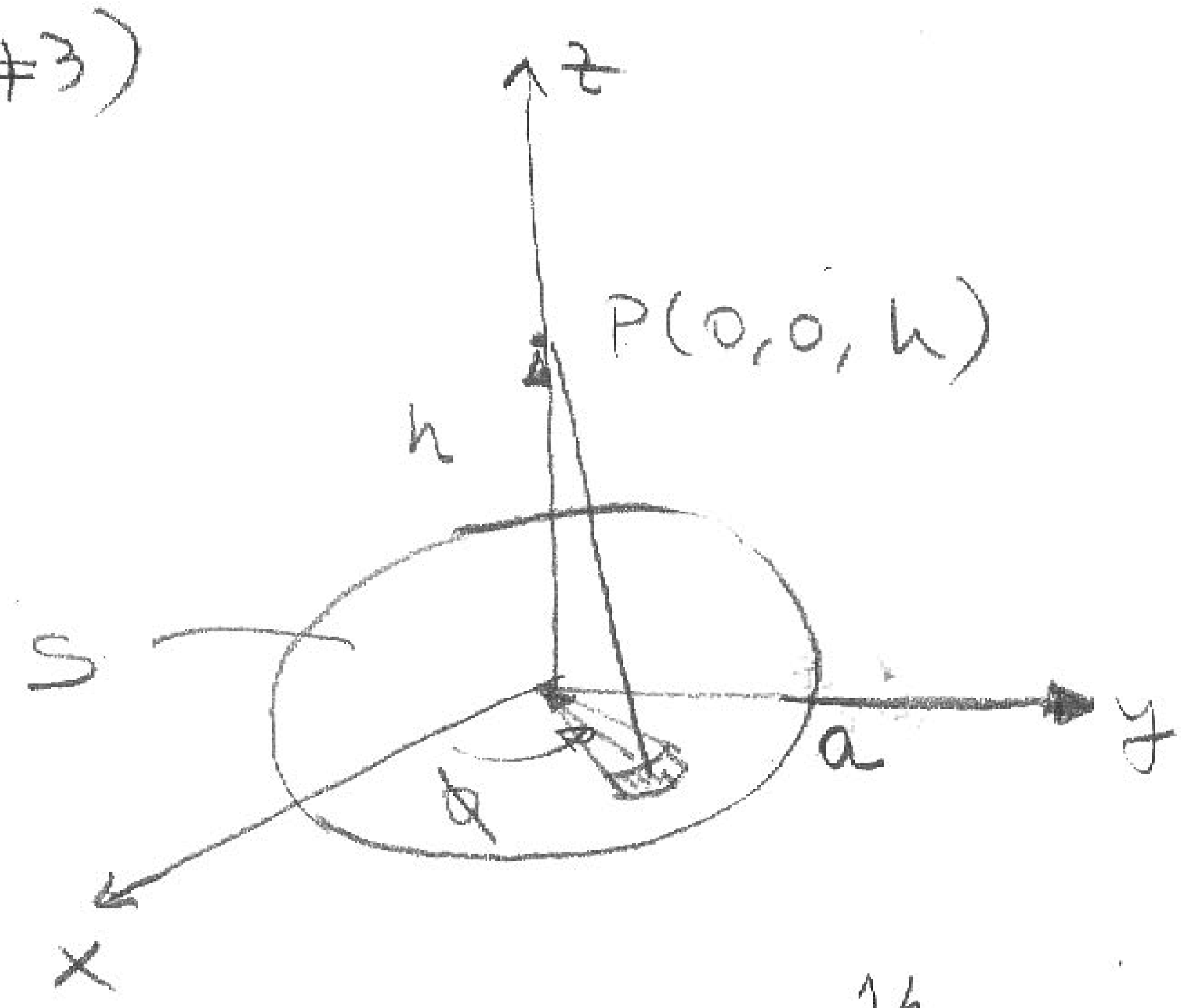
$$\vec{E}_L = \frac{10\pi\cancel{10^{-9}}}{\cancel{2\pi}\frac{\cancel{10^{-9}}}{\cancel{36}\pi}\sqrt{2}}\frac{1}{\sqrt{2}}(\vec{a}_x + \vec{a}_z) \doteq \frac{180\pi}{2}(\vec{a}_x + \vec{a}_z) = 90\pi(\vec{a}_x + \vec{a}_z)$$

18

$$\vec{E} = 180\pi\,\vec{a}_x + 360\pi\,\vec{a}_y + 90\pi\,\vec{a}_z \ \ V/m$$

$$\vec{E} = 565.48\,\vec{a}_x + 1130.97\,\vec{a}_y + 282.74\,\vec{a}_z \ \ V/m$$

#3)



$$V_s = \int_S \frac{\rho \, d\phi \, d\rho \, \rho_s}{4\pi\varepsilon_0 \left| -\rho \vec{a}_\rho + h \vec{a}_z \right|}$$

$$V_s = \frac{\rho_s}{4\pi\varepsilon_0} \int_S \frac{\rho \, d\rho \, d\phi}{(\rho^2 + h^2)^{1/2}}$$

$$V_s = \frac{\rho_s}{4\pi\varepsilon_0} \frac{(\rho^2+h^2)^{1/2}}{1/2} \Big|_0^a \cdot \phi \Big|_0^{2\pi} = \frac{\rho_s \cancel{2\pi}}{\cancel{2\pi} \varepsilon_0} \left(\sqrt{a^2+h^2} - h \right)$$

$$V_s = \frac{\rho_s}{\varepsilon_0} \left(\sqrt{a^2+h^2} - h \right)$$

$$V_Q = \frac{Q}{4\pi\varepsilon_0 \cdot h}$$

$$V = \frac{\rho_s}{\cancel{\varepsilon_0}} \left(\sqrt{h^2+a^2} - h \right) + \frac{Q}{4\pi \cancel{\varepsilon_0} h} = 0$$

$$\rho_s = -\frac{Q}{4\pi h \left(\sqrt{h^2+a^2} - h \right)} \quad C/m^2$$