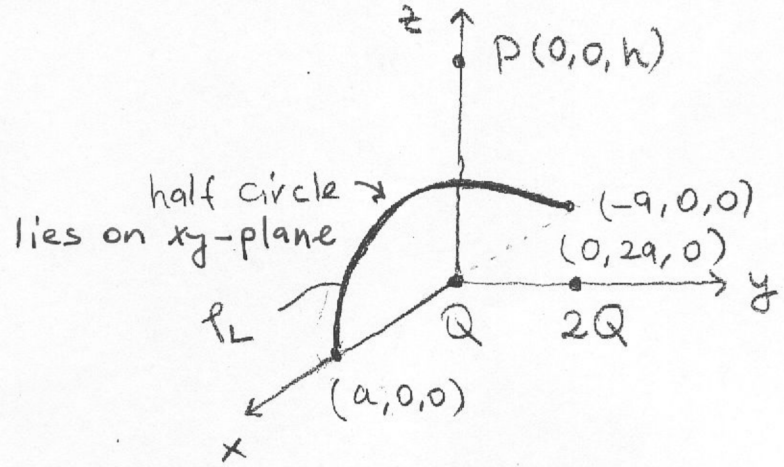
ELECTROMAGNETICS – I SECOND EXAM

Dr. Salih FADIL **August 10, 2010**

#1) A point charge is placed at the origin, calculate the energy stored in region $r > a$

#2) Consider the charge distribution shown in the figure. Calculate the potential at point $P(0,0,h)$. If this potential will be made zero, find the ρ_L in terms of Q.



#3) A point charge 100 pC is located at (5, 2, -4) while the x-axis carries charge $3\,nC/m$. If $z = 4$ plane carries charge $6\,nC/m^2$, find $\vec{E}$ at (1, 1, 1).

GOOD LUCK.... ☺

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SOLUTION MANUAL

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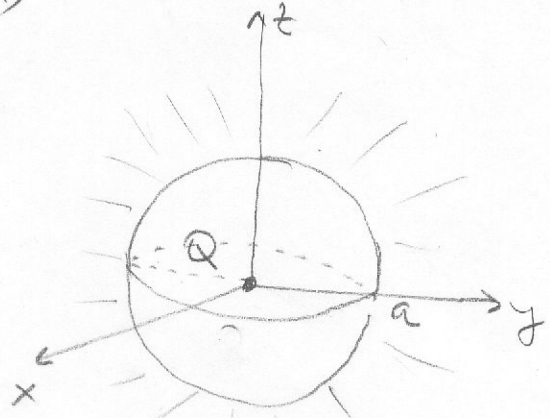
#1)



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

$$|\vec{E}| = \frac{Q}{4\pi\varepsilon_0 r^2}, \quad dv = r^2 \sin\theta \, d\theta \, d\phi \, dr$$

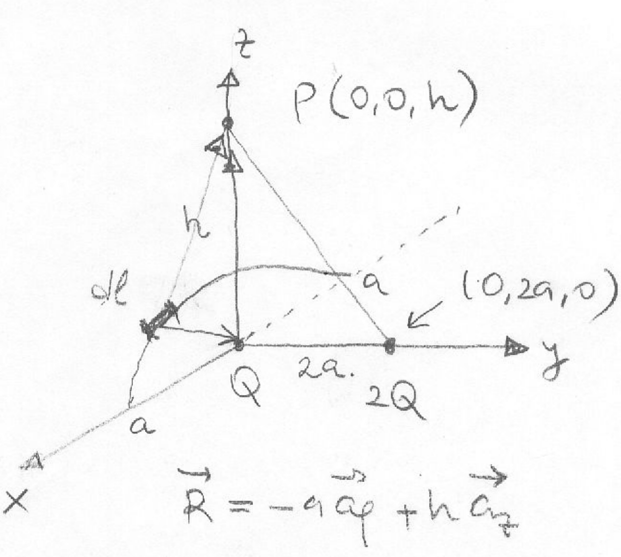
$$W_E = \frac{1}{2}\int_v \varepsilon_0 |\vec{E}|^2 dv$$

$$W_E = \frac{\varepsilon_0}{2}\int_v \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2 \cancel{r^2} \sin\theta \, d\theta \, d\phi \, dr = \frac{Q^2}{32\pi^2\varepsilon_0}\left\{\int_{r=a}^{\infty}\frac{dr}{r^2}\int_{\theta=0}^{\pi}\sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi\right\}$$

$$W_E = \frac{Q^2}{32\pi^2\varepsilon_0}\left\{\left(-\frac{1}{r}\right)\Big|_a^{\infty} \cdot (-\cos\theta)\Big|_0^{\pi} \cdot 2\pi\right\}$$

$$W_E = \frac{Q^2}{32\pi^2\varepsilon_0}\left(-0+\frac{1}{a}\right)\overbrace{(1+1)}^{2}2\pi = \frac{Q^2 \cancel{4\pi}}{\underbrace{\cancel{32\pi^2}}_{8\pi} a\varepsilon_0} = \frac{Q^2}{8\pi a \varepsilon_0} \text{ J.}$$

#2)



$$\vec{R} = -a\vec{a}_\rho + h\vec{a}_z$$

$$V_P = V_{\rho_L} + V_Q + V_{2Q}$$

$$V_Q = \frac{Q}{4\pi\varepsilon_0 h}$$

$$V_{2Q} = \frac{2Q}{4\pi\varepsilon_0 (h^2+4a^2)^{1/2}}$$

$$V_{\rho_L} = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L\, d\ell'}{R} = \frac{\rho_L}{4\pi\varepsilon_0}\int_L \frac{a\, d\phi'}{(a^2+h^2)^{1/2}}$$

$$V_{\rho_L} = \frac{\rho_L\, a}{4\pi\varepsilon_0 (a^2+h^2)^{1/2}}\, \phi'\Big|_{\pi}^{2\pi} = \frac{\rho_L\, a\, \cancel{\pi}}{4\cancel{\pi}\,\varepsilon_0 (a^2+h^2)^{1/2}} = \frac{a\, \rho_L}{4\varepsilon_0 (a^2+h^2)^{1/2}}$$

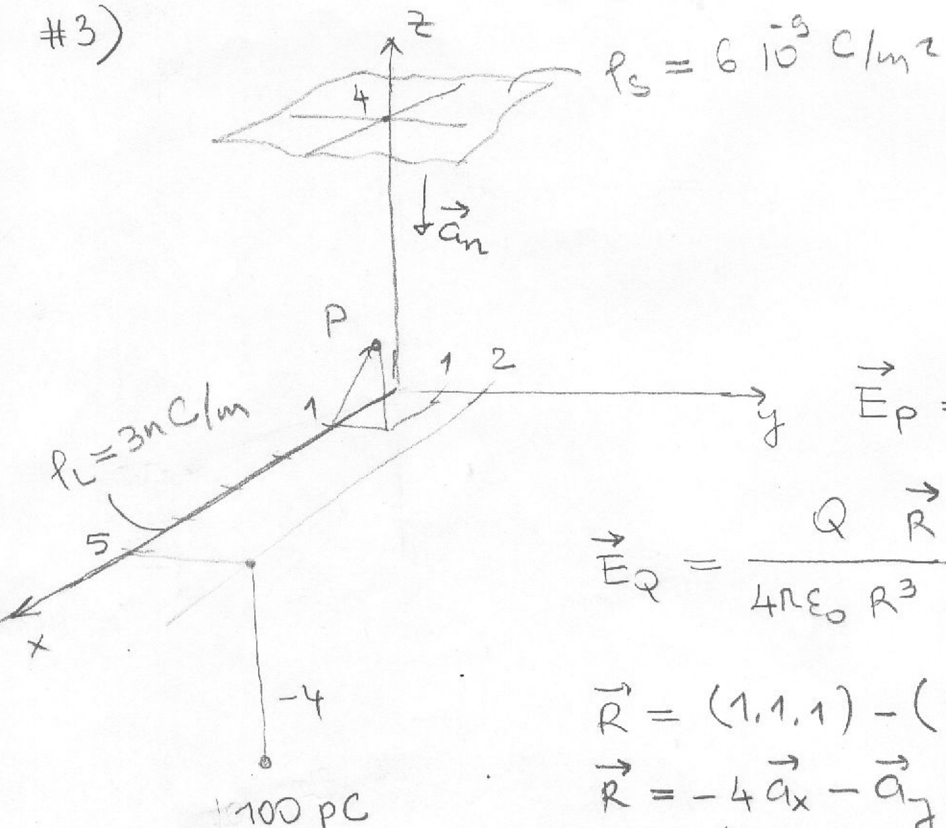
$$V_p = \frac{Q}{4\pi\varepsilon_0 h} + \frac{2Q}{4\pi\varepsilon_0 (h^2+4a^2)^{1/2}} + \frac{\pi a \rho_L}{4\pi\varepsilon_0 (a^2+h^2)^{1/2}}$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{h} + \frac{2Q}{\sqrt{h^2+4a^2}} + \frac{\pi a \rho_L}{(a^2+h^2)^{1/2}}\right]$$

$$V_p = 0 \qquad -Q\left[\frac{1}{h} + \frac{2}{\sqrt{h^2+4a^2}}\right] = \rho_L \frac{\pi a}{\sqrt{a^2+h^2}}$$

$$\rho_L = -\frac{\sqrt{a^2+h^2}\, Q}{\pi a}\left[\frac{1}{h} + \frac{2}{\sqrt{h^2+4a^2}}\right] = \frac{-Q}{\pi a}\left[\frac{\sqrt{a^2+h^2}}{h} + \frac{2\sqrt{h^2+a^2}}{\sqrt{h^2+4a^2}}\right]$$

#3)



$$\vec{E}_P = \vec{E}_Q + \vec{E}_{\rho_L} + \vec{E}_{\rho_s}$$

$$\vec{E}_Q = \frac{Q\ \vec{R}}{4\pi\varepsilon_0 R^3}$$

$$\vec{R} = (1,1,1) - (5,2,-4) = (-4,-1,5)$$

$$\vec{R} = -4\vec{a}_x - \vec{a}_y + 5\vec{a}_z$$

$$|R|^3 = (16+1+25)^{3/2} = (42)^{3/2}$$

(3)

$$\vec{E}_Q = \frac{Q}{4\pi\varepsilon_0 (42)^{3/2}}(-4\vec{a}_x - \vec{a}_y + 5\vec{a}_z)$$

$$\vec{E}_{\rho_L} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a}_\rho$$

$$\vec{\rho} = (1,1,1)-(1,0,0) = (0,1,1)$$

$$\vec{\rho} = \vec{a}_y + \vec{a}_z, \quad \rho = \sqrt{1+1} = \sqrt{2}$$

$$\vec{a}_\rho = \frac{1}{\sqrt{2}}(\vec{a}_y + \vec{a}_z)$$

$$\vec{E}_{\rho_L} = \frac{\rho_L}{2\pi\varepsilon_0\sqrt{2}}\,\frac{\vec{a}_y + \vec{a}_z}{\sqrt{2}} = \frac{\rho_L}{2\pi\varepsilon_0}\,\frac{\vec{a}_y + \vec{a}_z}{2}$$

$$\vec{E}_{\rho_S} = \frac{\rho_S}{2\varepsilon_0}\vec{a}_n \qquad \vec{a}_n = -\vec{a}_z$$

$$\vec{E}_{\rho_S} = \frac{6\,\cancel{10^{-9}}}{2\,\frac{\cancel{10^{-9}}}{36\pi}}(-\vec{a}_z) = \frac{\overset{3}{\cancel{6}}\times 36\pi}{\cancel{2}}(-\vec{a}_z) = \underline{\underline{-108\pi\,\vec{a}_z}}\ \text{V/m}$$

$$\vec{E}_{\rho_L} = \frac{3\,\cancel{10^{-9}}}{\cancel{2\pi}\,\frac{\cancel{10^{-9}}}{\underset{\underset{9}{\cancel{18}}}{\cancel{36\pi}}}\,\cancel{2}}(\vec{a}_y + \vec{a}_z) = \underline{\underline{27(\vec{a}_y + \vec{a}_z)}}\ \text{V/m}$$

$$\vec{E}_Q = \frac{100\,10^{-12}}{\cancel{4\pi}\,\frac{10^{-9}}{\underset{9}{\cancel{36\pi}}}(42)^{3/2}}(-4\vec{a}_x - \vec{a}_y + 5\vec{a}_z)$$

$$= \frac{900\,10^{9}\,10^{-12}}{(42)^{3/2}}(-4\vec{a}_x - \vec{a}_y + 5\vec{a}_z)$$

$$\vec{E}_Q = \underline{-0.013226\,\vec{a}_x - 0.003306\,\vec{a}_y + 0.016532\,\vec{a}_z}\ \text{V/m}$$

Finally

$$\vec{E}_P = -0.013226\,\vec{a}_x + 26.9967\,\vec{a}_y - 312.275\,\vec{a}_z\ \text{V/m}$$