ELECTROMAGNETICS – I SECOND MIDTERM EXAM

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#1) The line $y=1, z=-4$ carries charge $30\,nC/m$ while planes $x=1$ and $z=-2$ carry charges 20 nC/m^2 and $-30\,nC/m^2$ respectively. A point charge of $100\,nC$ is located at $(0,5,0)$. Calculate the $\vec{E}$ value at the origin.

#2) In electric field $\vec{E}=20r\sin\theta\,\vec{a}_r+10r\cos\theta\,\vec{a}_\theta\ V/m$, calculate the work done in transferring a $10\,nC$ charge from $A(5,30^\circ,0^\circ)$ to $B(10,90^\circ,60^\circ)$. Who does the work?

#3) If $\vec{D}=2z^2\sin\frac{\phi}{2}\vec{a}_\rho+z^2\cos\frac{\phi}{2}\vec{a}_\phi+4z\rho\sin\frac{\phi}{2}\vec{a}_z\ mC/m^2$, using Gauss's law determine the total charge enclosed by the object defined $-2\le z\le 1,\quad 1\le\rho\le 4,\quad 0\le\phi\le 2\pi$.

GOOD LUCK...☺

ELECTROMAGNETICS – I SECOND MIDTERM EXAM SOLUTION MANUAL

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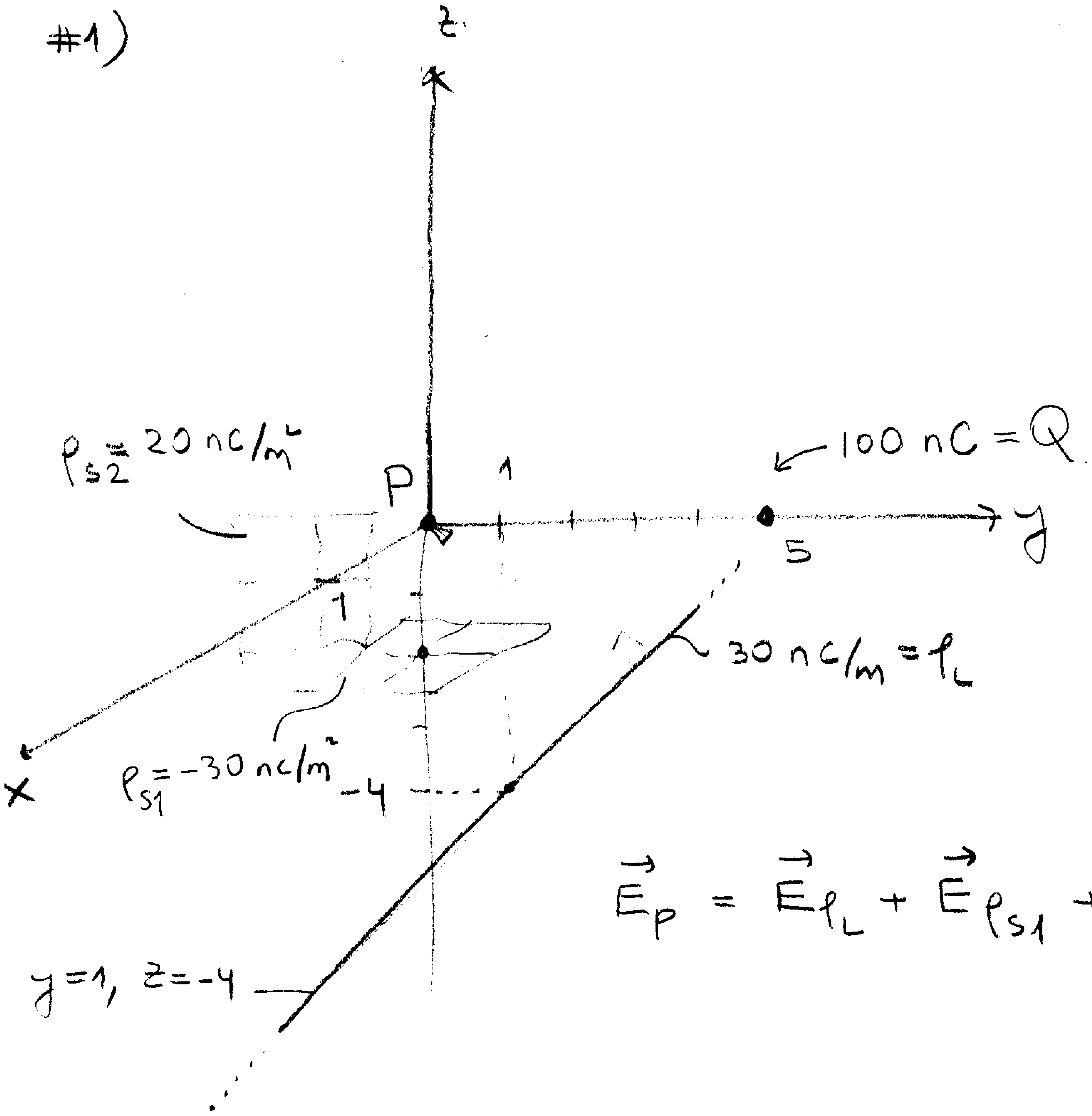
#1)



$$\vec{E}_P = \vec{E}_{\rho_L} + \vec{E}_{\rho_{s1}} + \vec{E}_{\rho_{s2}} + E_Q$$

$$\vec{E}_{\rho_L} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a}_\rho$$

$$\vec{\rho} = (0,0,0) - (0,1,-4) = (0,-1,4)$$

$$|\vec{\rho}| = \rho = (1+16) = \sqrt{17}$$

$$\vec{a}_\rho = \frac{-a_y + 4\vec{a}_z}{\sqrt{17}}$$

$$\vec{E}_{\rho_L} = \frac{30\,\cancel{10^{-9}}}{\cancel{2\pi}\,\frac{\cancel{10^{-9}}}{\cancel{36\pi}_{18}}\sqrt{17}}\;\frac{-a_y + 4\vec{a}_z}{\sqrt{17}} = \frac{18\times 30}{17}(-\vec{a}_y + 4\vec{a}_z)$$

$$= \underline{\underline{-31.764\,\vec{a}_y + 127.05\,\vec{a}_z}} \text{ V/m}$$

$$\vec{E}_{\rho s} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n \;;\quad \vec{E}_{\rho s_1} = \frac{-30\,\cancel{10^{-9}}}{\cancel{2}\,\frac{\cancel{10^{-9}}}{\cancel{36\pi}\;18}}\cdot\vec{a}_z = \underline{-1696.46\,\vec{a}_z\ \text{V/m}}$$

$$\vec{E}_{\rho s_2} = \frac{20\,\cancel{10^{-9}}}{\cancel{2}\,\frac{\cancel{10^{-9}}}{\cancel{36\pi}\;18\pi}}(-\vec{a}_x) = -1130.97\,\vec{a}_x\ \text{V/m.}$$

$$\vec{E}_Q = \frac{Q\,\vec{R}}{4\pi\varepsilon_0 R^3} = \frac{100\,\cancel{10^{-9}}(-\cancel{5}\,\vec{a}_y)}{\cancel{4\pi}\,\frac{\cancel{10^{-9}}}{\cancel{36\pi}\;9}\,\cancel{(5)^3}\;5^2} = -\frac{900\,\vec{a}_y}{25} = -36\,\vec{a}_y$$

$$\vec{E}_p = -1130.97\,\vec{a}_x + (-31.764-36)\vec{a}_y + (127.05-1696.46)\vec{a}_z\ \text{V/m}$$

$$\boxed{\vec{E}_p = -1130.97\,\vec{a}_x - 67.764\,\vec{a}_y - 1569.55\,\vec{a}_z\ \text{V/m}}$$

#2) Since the work done is independent of the followed trajectory in the elektrik field, the trajectory can be selected as follows

$$A(5,30^\circ,0) \longrightarrow C(10,30^\circ,0^\circ) \longrightarrow D(10,90^\circ,0^\circ) \longrightarrow B(10,90^\circ,15^\circ)$$

$$\vec{dl} = dr\,\vec{a}_r \qquad \vec{dl} = r\,d\theta\,\vec{a}_\theta \qquad \vec{dl} = r\sin\theta\,d\phi\,\vec{a}_\phi$$

$$W = -Q\int_A^B \vec{E}\cdot\vec{dl} = -10\,10^{-9}\left\{\int_A^C + \int_C^D + \int_D^B\right\}\vec{E}\cdot\vec{dl}$$

$$\int_A^C E\,\vec{dl} = \int_{r=5}^{10} 20\,r\sin\theta\,dr = 20\,\underbrace{\sin(30^\circ)}_{0.5}\,\frac{r^2}{2}\Big|_5^{10}$$

$$= \frac{10}{2}(100-25) = 375$$

$$\int_C^D \vec{E}\cdot\vec{dl} = \int_{30^\circ}^{90^\circ} r\,d\theta\ 10\,r\cos\theta = 10\,(10)^2 \int_{30^\circ}^{90^\circ} \cos\theta\,d\theta$$

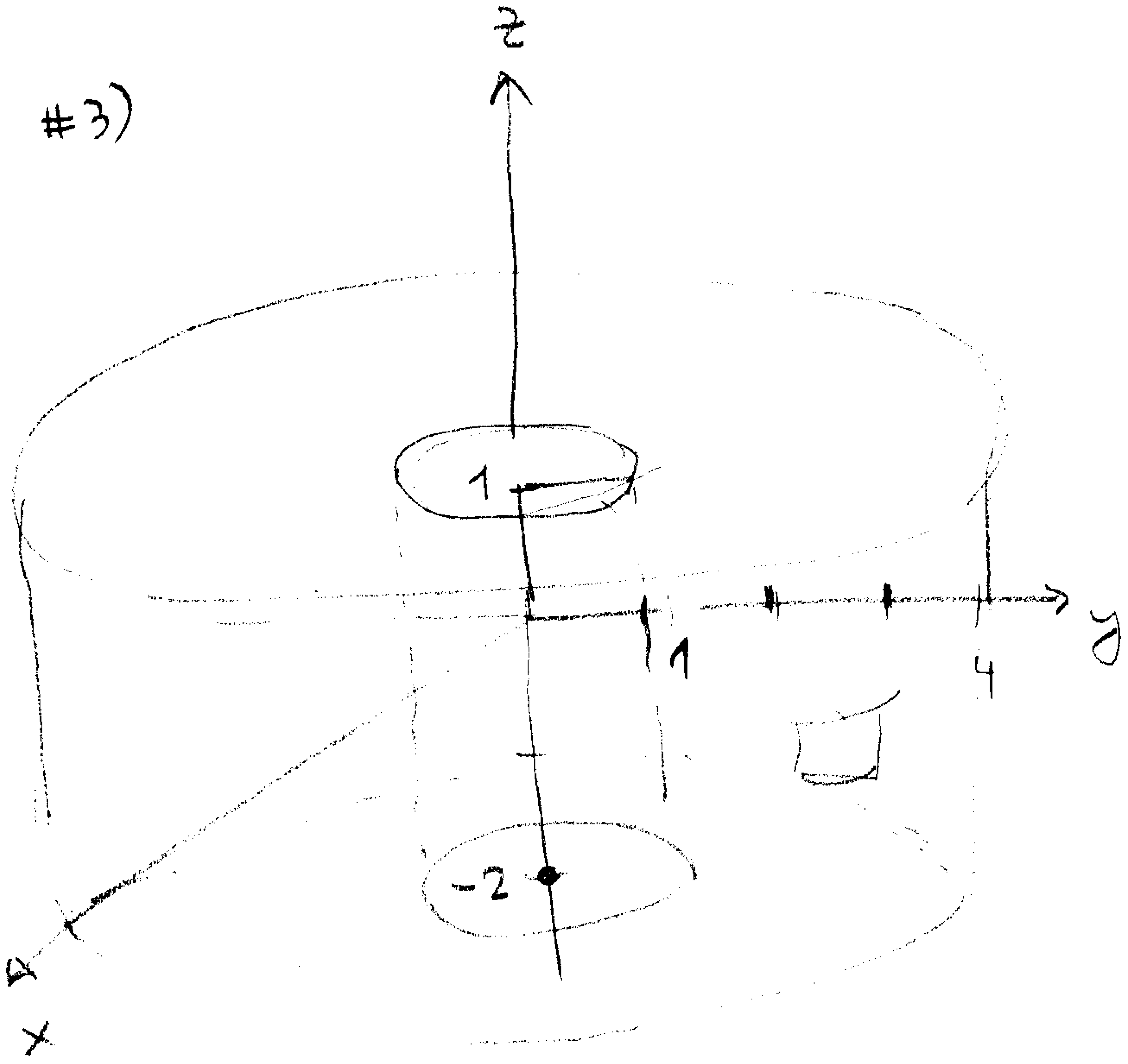
$$= 1000\ \sin\theta \Big|_{30^\circ}^{90^\circ} = 1000\,(1-0.5) = 500$$

$$\int_D^B \vec{E}\cdot\vec{dl} = \int_D^B \cdot\, 0 = 0.$$

$$W = -10\ 10^{-9}\left\{375 + 500 + 0\right\} = -8750\ 10^{-9}\ J.$$
$$= -8.75\ \mu J$$

Since $W<0$, the field does the work!

#3)



Gauss law

$$\oint_S \vec{D}\cdot\vec{dS} = Q_{enc}$$

$$\oint_S \vec{D}\cdot\vec{dS} = \Big(\underbrace{\int}_{\text{top}} + \underbrace{\int}_{\text{bottom}} + \underbrace{\int}_{\substack{\text{curved}\\\text{inner}}} + \underbrace{\int}_{\substack{\text{curved}\\\text{outer}}}\Big)\vec{D}\cdot\vec{dS} = Q_{enc}$$

Top surface

$\vec{dS} = \rho \, d\phi \, d\rho \, \vec{a_z}$; $z = 1$

$$\int_{top} \vec{D} \cdot \vec{dS} = \int_{top} 4z\rho \sin\frac{\phi}{2} \, \rho \, d\phi \, d\rho = 4 \int_{\rho=1}^{4} \rho^2 d\rho \cdot \int_{\phi=0}^{2\pi} \sin\frac{\phi}{2} \, d\phi$$

$$= 4 \left.\frac{\rho^3}{3}\right|_1^4 \cdot \left. 2\left(-\cos\frac{\phi}{2}\right)\right|_0^{2\pi} = \frac{4}{3}(64-1) \, 2 \underbrace{\left(-\cos\pi + \cos 0\right)}_{2}$$

$$= \frac{4 \times 63}{3} \, 4 = 336$$

Bottom surface

$\vec{dS} = \rho \, d\phi \, d\rho \, (-\vec{a_z})$; $z = -2$

$$\int_{bottom} \vec{D} \cdot \vec{dS} = -\int_{bottom} 4z\rho \sin\frac{\phi}{2} \, \rho \, d\rho \, d\phi = 8 \int_{\rho=1}^{4} \rho^2 d\rho \int_{\phi=0}^{2\pi} \sin\frac{\phi}{2} \, d\phi$$

$$= 8 \left(\frac{64-1}{3}\right) 4 = 672$$

Outer curved surface

$\vec{dS} = \rho \, d\phi \, dz \, \vec{a_\rho}$; $\rho = 4$

$$\int_{\substack{curved \\ outer}} \vec{D} \cdot \vec{dS} = \int_{\substack{curved \\ outer}} 2z^2 \sin\frac{\phi}{2} \cdot \rho \, d\phi \, dz = 8 \int_{z=-2}^{1} z^2 \, dz \int_{\phi=0}^{2\pi} \sin\frac{\phi}{2} \, d\phi$$

$$= 8 \left.\frac{z^3}{3}\right|_{-2}^{1} \left. 2\left(-\cos\frac{\phi}{2}\right)\right|_{\phi=0}^{2\pi} = \frac{8}{3}(1+8) \, 2 \, (1+1)$$

$$= \frac{8 \times 9 \times 2 \times 2}{3} = 96$$

Inner curved surface

$\vec{dS} = \rho \, d\phi \, dz \, (-\vec{a_\rho})$; $\rho = 1$

$$\int_{\substack{curved \\ outer}} \vec{D} \, \vec{dS} = -\int_{\substack{curved \\ outer}} 2z^2 \sin\frac{\phi}{2} \, \rho \, d\phi \, dz = -2 \int_{z=-2}^{1} z^2 \, dz \int_{\phi=0}^{2\pi} \sin\phi/2 \, d\phi$$

$$= -2 \, \frac{1}{3}(1+8) \, 2 \cdot 2 = -8 \, \frac{9}{3} = -24$$

$$Q_{enc} = (336 + 672 + 96 - 24) \, mC = \underline{\underline{1080 \, mC}}$$