ELECTROMAGNETICS I SECOND MIDTERM EXAM

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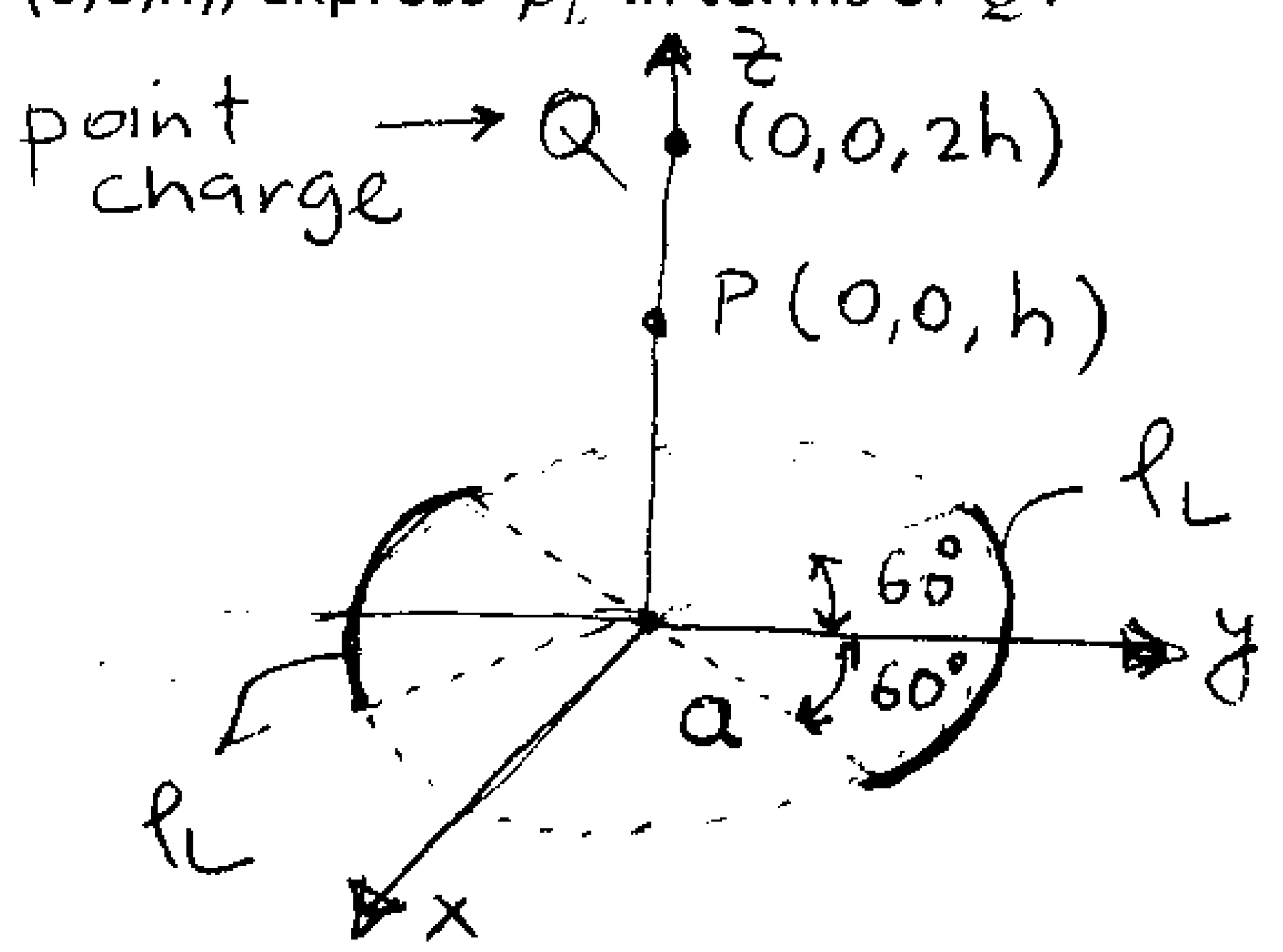
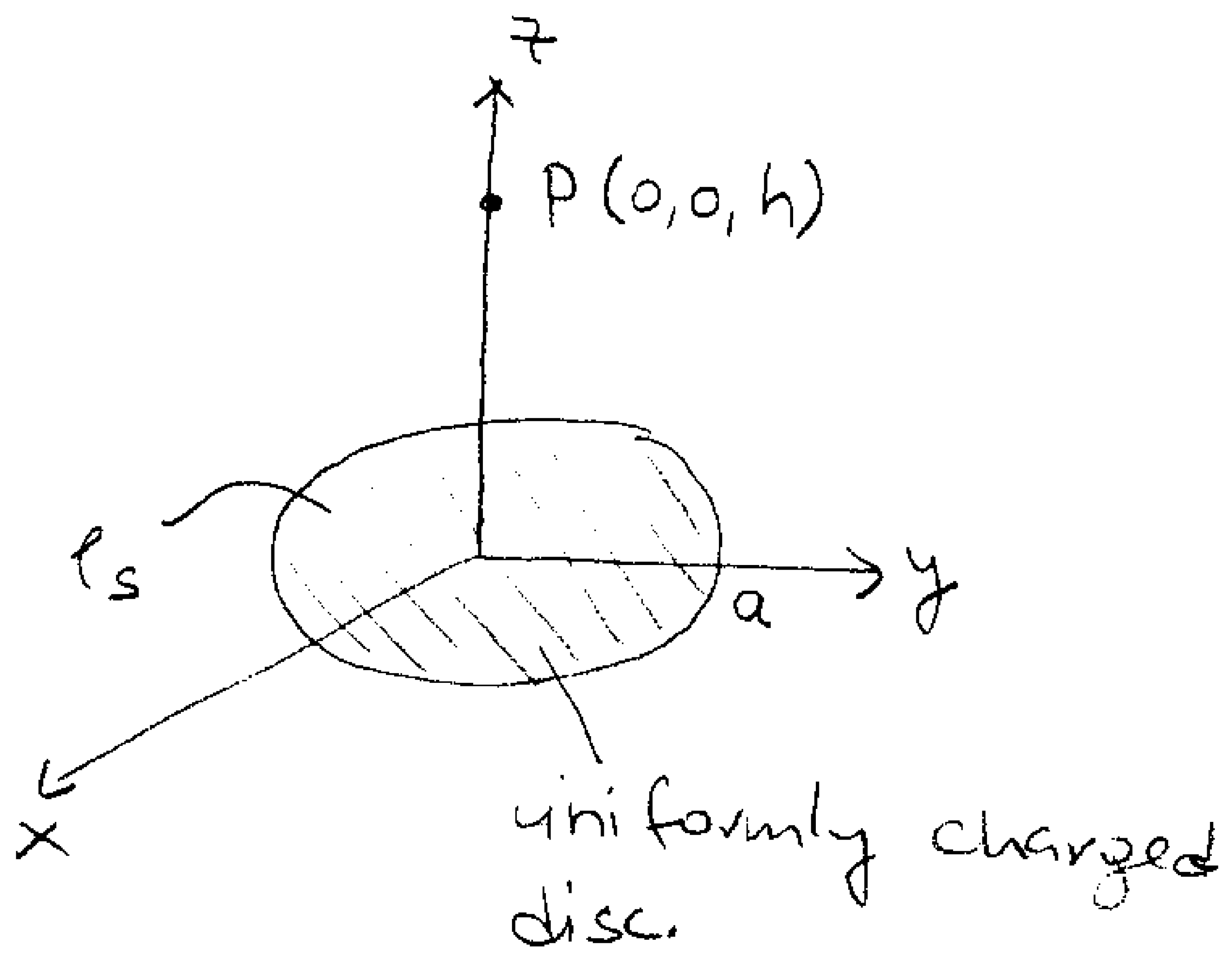
#1) Consider the figure given in the following. If the electric field intensity vector becomes zero at point P(0,0,h), express ρ_L in terms of Q.



#2) If plane $x = 3$ carry charge 20 nC/m^2, lines $x = 5, z = 3$ and $x = 7, y = 6$ carry charges 30 nC/m and -25 nC/m respectively, 50 nC point charge at pint (0, 0, -5), calculate the $\vec{E}$ value at point (1, 1, 1).

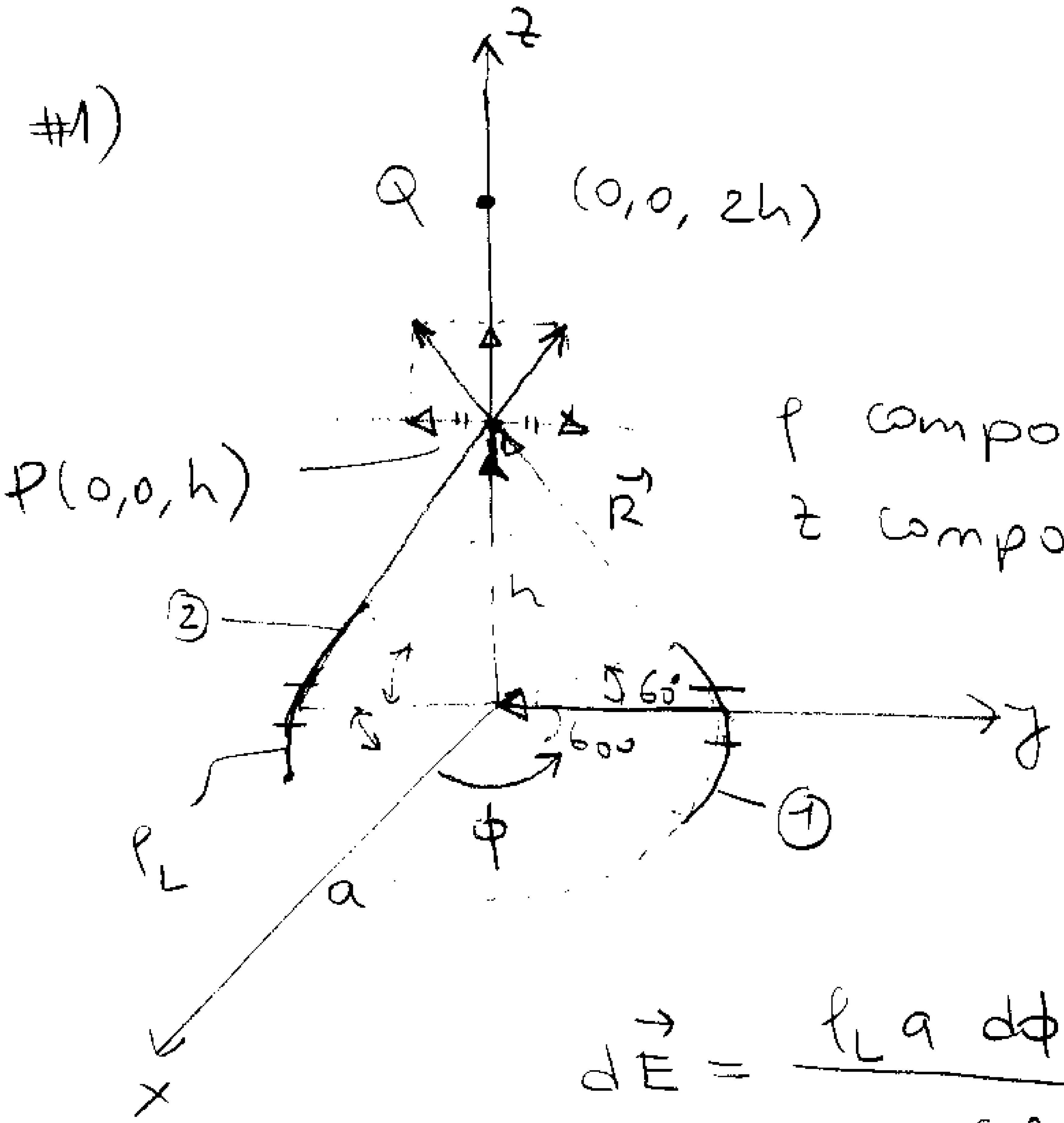
#3) Consider the figure given in the following. Calculate $\vec{E}$ at point P.



Good luck...☺

ELECTROMAGNETICS-I SECOND MIDTERM EXAM SOLUTION MANUAL

#1)



ρ components cancel each other
z components are added up.

$$\vec{R} = -a\vec{a}_\rho + h\vec{a}_z$$

$$dQ = a\, d\phi\, \rho_L$$

$$d\vec{E} = \frac{\rho_L a\, d\phi}{4\pi\varepsilon_0 (a^2+h^2)^{3/2}} (-a\vec{a}_\rho + h\vec{a}_z)$$

$$\vec{E}_1 = \frac{\rho_L a}{4\pi\varepsilon_0 (a^2+h^2)^{3/2}} \int_{\phi=30°}^{150°} \left\{ -a\left(\cos\phi\, \vec{a}_x + \sin\phi\, a_y\right) + h\vec{a}_z \right\} d\phi$$

$$\vec{E}_2 = \frac{\rho_L a}{4\pi\varepsilon_0 (\rho^2+h^2)^{3/2}} \int_{\phi=210}^{330°} \left\{ (-a\cos\phi\, \vec{a}_x + \sin\phi\, \vec{a}_y) + h\vec{a}_z \right\} d\phi$$

$$\vec{E}_L = \vec{E}_1 + \vec{E}_2 = \frac{\rho_L a}{4\pi\varepsilon_0 (\rho^2+h^2)^{3/2}} \left\{ -a\left(\sin\phi \Big|_{30°}^{150°} \vec{a}_x - \cos\phi \Big|_{30°}^{150°} \vec{a}_y \right) + h\phi \Big|_{\pi/6}^{5\pi/6} \vec{a}_z \right.$$

$$\left. + -a\left(\sin\phi \Big|_{210°}^{330°} \vec{a}_x - \cos\phi \Big|_{210°}^{330°} \vec{a}_y \right) + h\phi \Big|_{7\pi/6}^{11\pi/6} \vec{a}_z \right\}$$

$$\vec{E}_L = \frac{\rho_L a}{4\pi\varepsilon_0 (\rho^2+h^2)^{3/2}} \Big\{ -a\Big[\left(\cancel{\sin 150^\circ} - \cancel{\sin 30^\circ} + \cancel{\sin 330^\circ} - \cancel{\sin 210^\circ}\right)\vec{a}_x$$

$$+ \left(\cancel{\cos 30^\circ} - \cancel{\cos 150^\circ} + \cancel{\cos 210^\circ} - \cancel{\cos 330^\circ}\right)\vec{a}_y +$$

$$+\Big[h\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + h\left(\frac{11\pi}{6} - \frac{7\pi}{6}\right)\Big]\vec{a}_z \Big\}$$

$$\vec{E}_L = \frac{\rho_L a}{4\pi\varepsilon_0 (\rho^2+h^2)^{3/2}} \left[\frac{2\pi}{3} + \frac{2\pi}{3}\right] h\,\vec{a}_z$$

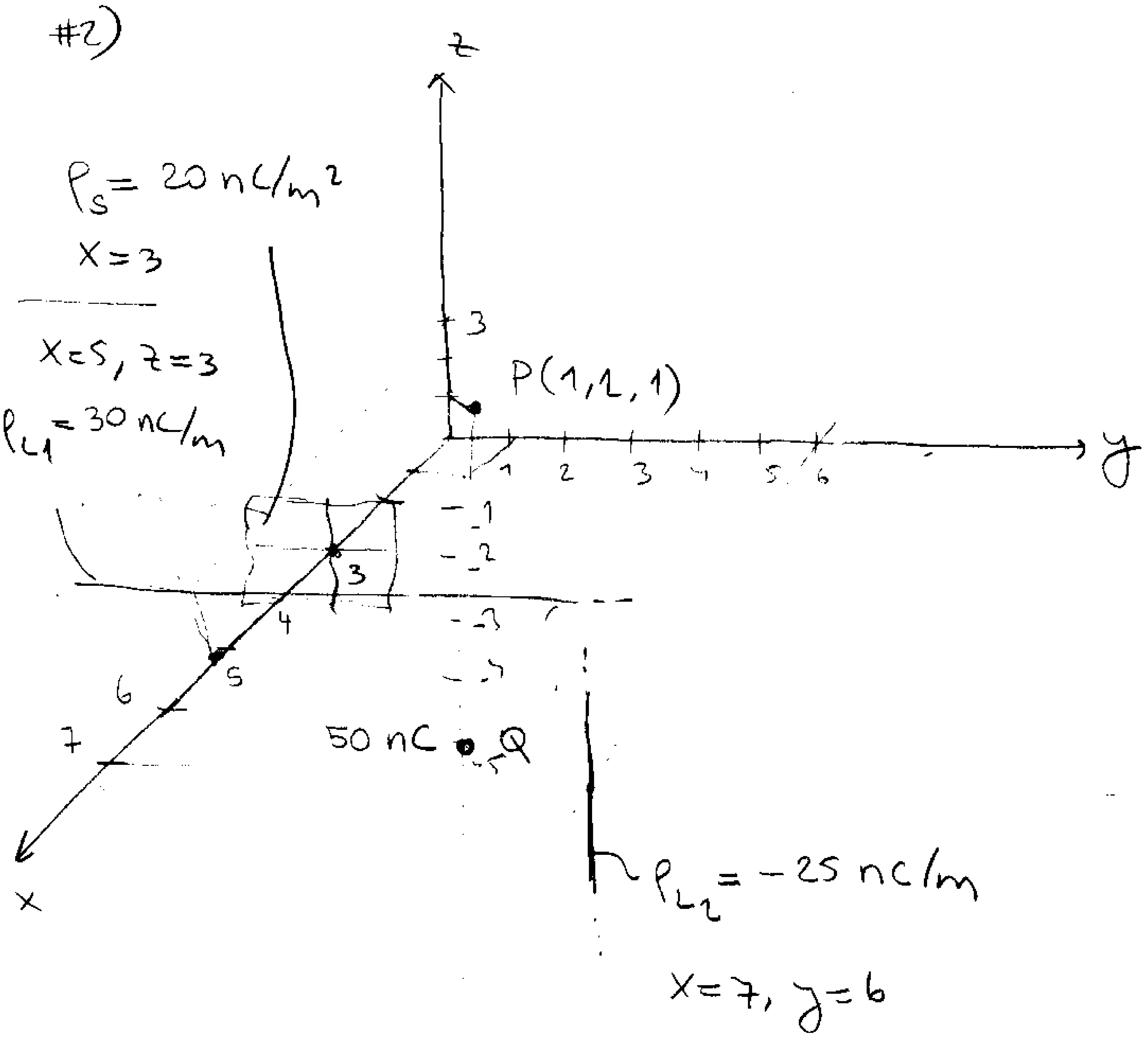
$$\vec{E}_L = \frac{\rho_L a}{\cancel{4\pi}\varepsilon_0 (\rho^2+h^2)^{3/2}} \frac{\cancel{4\pi}}{3} h\,\vec{a}_z = \frac{\rho_L a h}{3\varepsilon_0 (\rho^2+h^2)^{3/2}}\vec{a}_z$$

$$\vec{E}_Q = \frac{Q}{4\pi\varepsilon_0 . h^2}(-\vec{a}_z) \ \mathrm{V/m.}$$

$$\vec{E}_P = \underbrace{\left(\frac{\rho_L a . h}{3\varepsilon_0 (\rho^2+h^2)^{3/2}} - \frac{Q}{4\pi\varepsilon_0 h^2}\right)}_{=0} \vec{a}_z = 0$$

$$\rho_L = \frac{3Q(\rho^2+h^2)^{3/2}}{4\pi h^2 a h} \qquad \boxed{\rho_L = \frac{3Q(\rho^2+h^2)^{3/2}}{4\pi a h^3}}$$

#2)



$$\vec{E}_P = \vec{E}_{L1} + \vec{E}_{L2} + \vec{E}_S + \vec{E}_Q$$

$$\vec{E}_{L1} = \frac{\rho_{L1}}{2\pi\varepsilon_0 \rho}\frac{\vec{\rho}}{\rho} \qquad \vec{\rho} = (1,1,1)-(5,1,3) = (-4,0,-2)$$

$$\rho = \sqrt{16+4} = \sqrt{20}$$

$$\vec{E}_{L1} = \frac{30\cancel{10^{-9}}}{\cancel{2\pi}\frac{\cancel{10^{-9}}}{\cancel{36\pi}}_{18}}\ \frac{-4\vec{a}_x - 2\vec{a}_z}{20} = \frac{30\times 18}{20}\left(-4\vec{a}_x - 2\vec{a}_z\right)$$

$$\boxed{\vec{E}_{L1} = -108\,\vec{a}_x - 54\,\vec{a}_z\ V/m}$$

$$\vec{E}_{L2} = \frac{\rho_{L2}}{2\pi\varepsilon_0 \rho}\frac{\vec{\rho}'}{\rho} \qquad \vec{\rho} = (1,1,1)-(7,6,1) = (-6,-5,0)$$

$$\rho = \sqrt{36+25} = \sqrt{61}$$

$$\vec{E}_{L2} = \frac{-25\cancel{10^{-9}}}{\cancel{2\pi}\frac{\cancel{10^{-9}}}{\cancel{36\pi}}_{18}}\ \frac{-6\vec{a}_y - 5\vec{a}_z}{61} = -\frac{18\times 25}{61}\left(-6\vec{a}_x - 5\vec{a}_y\right)$$

$$\boxed{\vec{E}_{L2} = +44.26\,\vec{a}_x + 36.88\,\vec{a}_y\ V/m}$$

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n = \frac{\cancel{20}\ \cancel{10^{-9}}}{\cancel{2}\ \frac{\cancel{10^{-9}}}{36\pi}}(-\vec{a}_x) = -360\pi\,\vec{a}_x \ \text{V/m}$$

$$\boxed{\vec{E}_s = -1130{,}9\ \vec{a}_x \ \text{V/m}}$$

$$\vec{E}_Q = \frac{Q\ \vec{R}}{4\pi\varepsilon_0 R^3}$$

$$\vec{R} = (1,1,1) - (0,0,-5) = (1,1,6)$$

$$R = (36+1+1)^{1/2} = \sqrt{38}$$

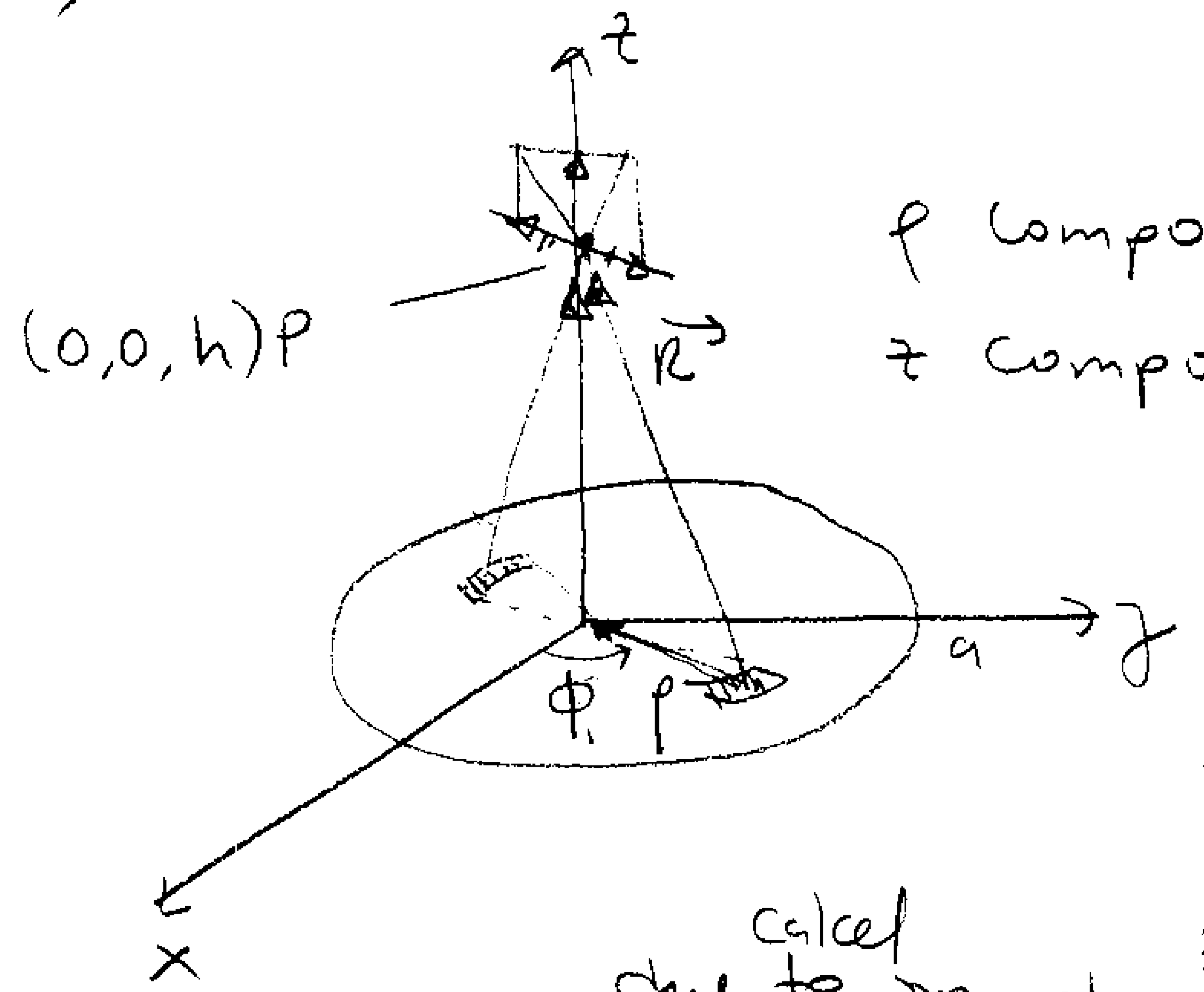
$$\vec{E}_Q = \frac{50\ \cancel{10^{-9}}}{\cancel{4\pi}\ \frac{\cancel{10^{-9}}}{\cancel{36\pi}_{9}}(38)^{3/2}}(\vec{a}_x + \vec{a}_y + 6\vec{a}_z) = \frac{450}{(38)^{3/2}}(1,\ 1,\ 6)$$

$$\boxed{\vec{E}_Q = 1{,}921\ \vec{a}_x + 1{,}921\ \vec{a}_y + 11{,}526\ \vec{a}_z} \ \text{V/m}$$

$$\vec{E}_P = (-108 + 44{,}26 - 1130{,}9)\vec{a}_x + (36{,}88 + 1{,}921)\vec{a}_y + (-54 + 11{,}526)\vec{a}_z$$

$$\boxed{\vec{E}_P = -1194{,}64\ \vec{a}_x + 38{,}801\vec{a}_y - 42{,}474}$$

#3)



ρ components cancel each other
z components add up.

$$dQ = \rho \, d\phi \, d\rho \, \rho_s$$

$$\vec{R} = -\rho \vec{a}_\rho + h \vec{a}_z$$

$$R = (\rho^2 + h^2)^{1/2}$$

calcel due to symmetry

$$d\vec{E} = \frac{\rho \, d\phi \, d\rho \, \rho_s}{4\pi\varepsilon_0} \; \frac{(\cancel{-\rho \vec{a}_\rho} + h \vec{a}_z)}{(\rho^2 + h^2)^{3/2}}$$

$$\vec{E} = \frac{\rho_s h \vec{a}_z}{4\pi\varepsilon_0} \left\{ \frac{1}{2} \int_{\rho=0}^{a} (\rho^2 + h^2)^{3/2} \, 2\rho \, d\rho \int_{\phi=0}^{2\pi} d\phi \right\}$$

$$= \frac{\rho_s h \vec{a}_z}{\cancel{4\pi}_{\,2} \varepsilon_0} \left\{ \cancel{\frac{1}{2}} \; \frac{+1}{\sqrt{\rho^2 + h^2}} \; \frac{1}{\cancel{(-1/2)}} \Big|_{\rho=0}^{a} \cdot \cancel{2\pi} \right\}$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0} \vec{a}_z \left[\frac{1}{h} - \frac{1}{\sqrt{h^2 + a^2}} \right]$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \left[1 - \frac{h}{\sqrt{h^2 + a^2}} \right] \vec{a}_z}$$

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