BALANCED THREE-PHASE CIRCUITS

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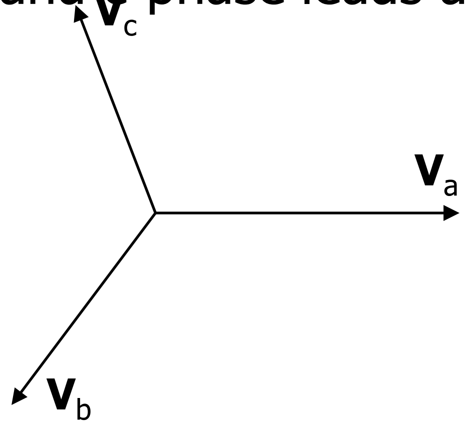
BALANCED THREE-PHASE VOLTAGES

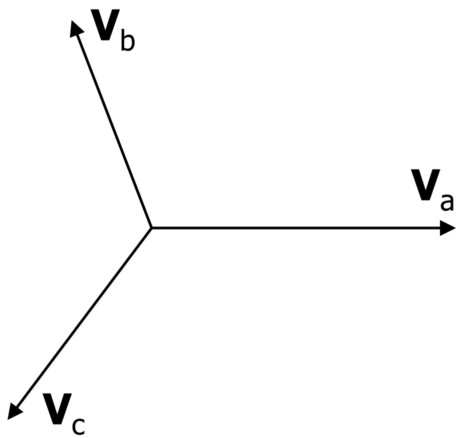
A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120^0. The phases are referred to as a, b, and c, and usually the a-phase is taken as the reference.

abc (positive) phase sequence: b-phase lags a-phase by 120^0,
and c-phase leads a-phase by 120^0.



$$\mathbf{V}_a = V_m \angle 0^0$$
$$\mathbf{V}_b = V_m \angle -120^0$$
$$\mathbf{V}_c = V_m \angle +120^0$$

acb (negative) phase sequence: b-phase leads a-phase by 120^0, and c-phase lags a-phase by 120^0.



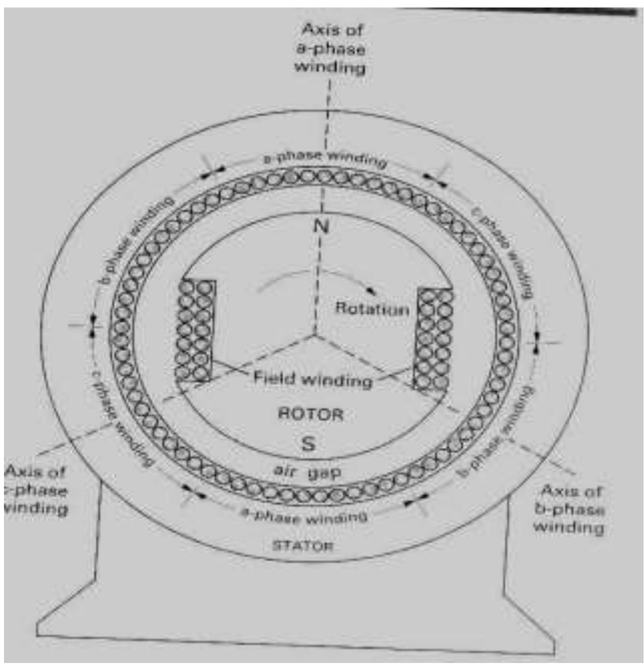
$$\mathbf{V}_a = V_m \angle 0^0$$
$$\mathbf{V}_b = V_m \angle +120^0$$
$$\mathbf{V}_c = V_m \angle -120^0$$

Another important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero.

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$$

THREE-PHASE VOLTAGE SOURCES

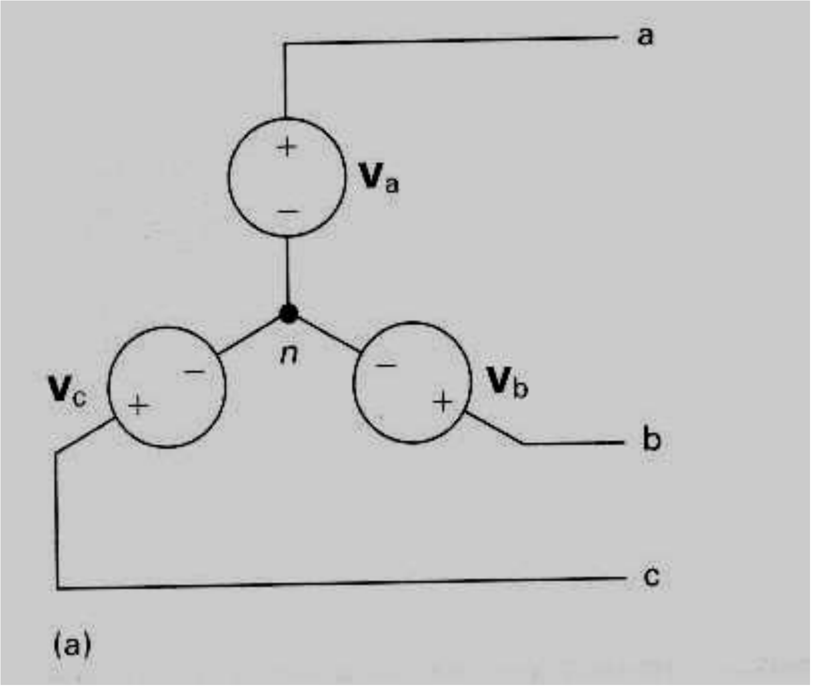
A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator.

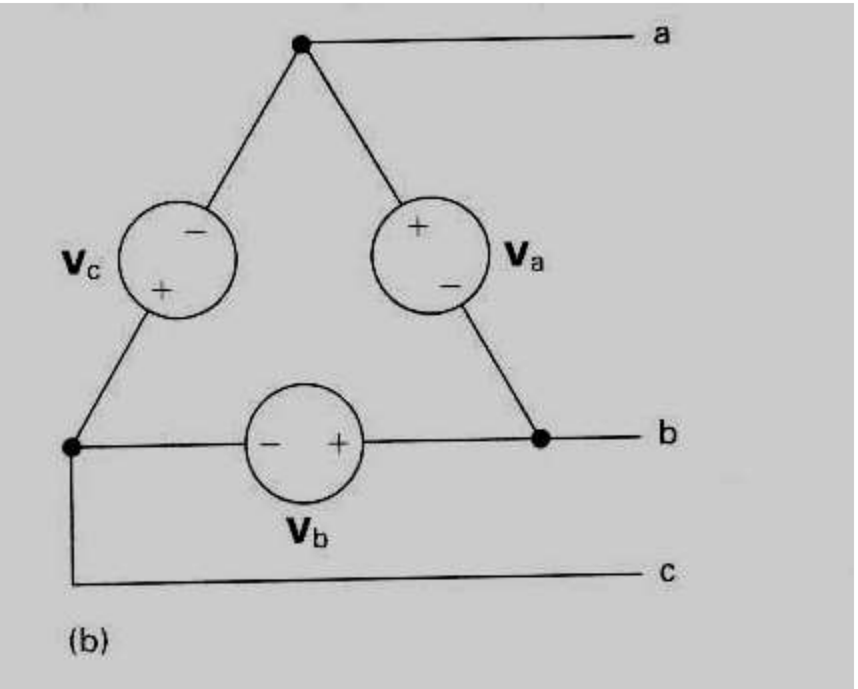


The rotor of the generator is an electro-magnet driven at synchronous speed by a prime mover. Rotation of the electro-magnet induces a sinusoidal voltage in each winding.

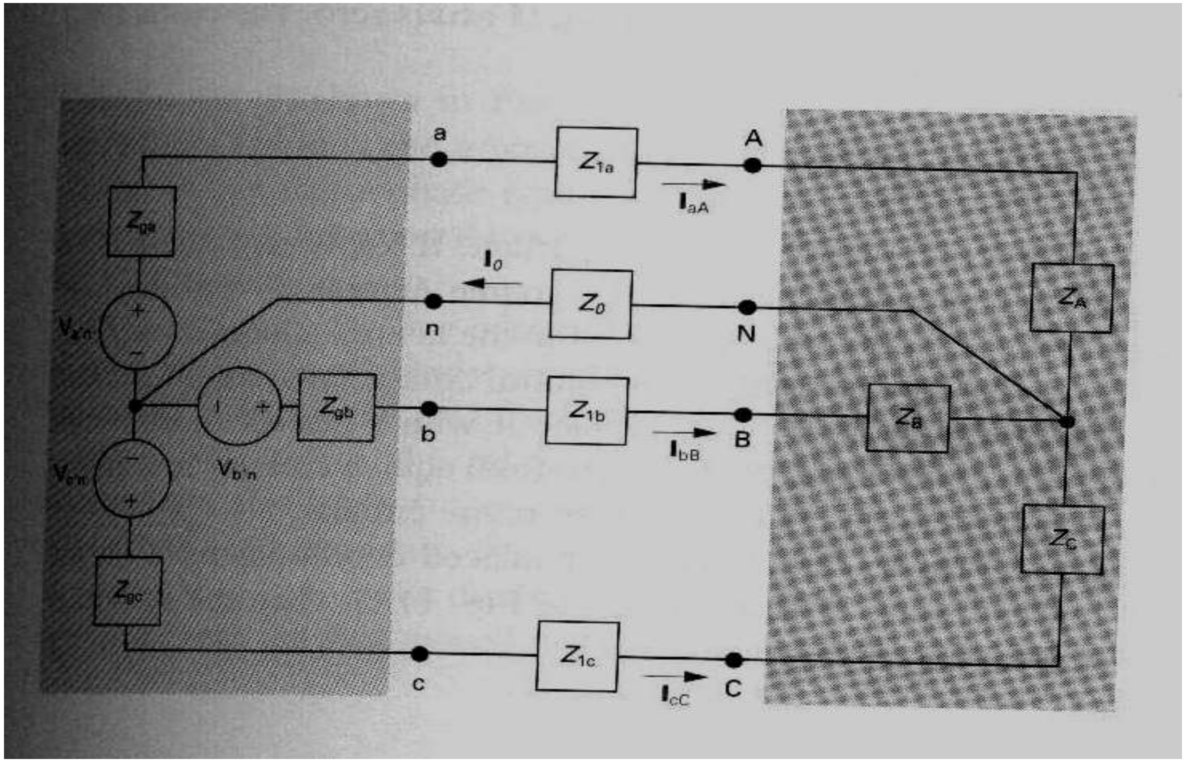
The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120^0. The phase windings are stationary with respect to the rotating electromagnet.

There are two ways of interconnecting the separate phase windings to form a 3-phase source: in either a wye (Y) or a delta (Δ).





ANALYSIS OF THE Y-Y CIRCUIT



$$\frac{\mathbf{V}_N}{Z_0}+\frac{\mathbf{V}_N-\mathbf{V}_{a'n}}{Z_A+Z_{1a}+Z_{ga}}+\frac{\mathbf{V}_N-\mathbf{V}_{b'n}}{Z_B+Z_{1b}+Z_{gb}}+\frac{\mathbf{V}_N-\mathbf{V}_{c'n}}{Z_C+Z_{1c}+Z_{gc}}=0$$

BALANCED THREE-PHASE CIRCUIT

- The voltage sources form a set of balanced three-phase voltages, this means $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, $\mathbf{V}_{c'n}$ are a set of balanced three-phase voltages.
- The impedance of each phase of the voltage source is the same: $Z_{ga}=Z_{gb}=Z_{gc}$.
- The impedance of each line conductor is the same: $Z_{1a}=Z_{1b}=Z_{1c}$.
- The impedance of each phase of the load is the same: $Z_A=Z_B=Z_C$
- There is no restriction on the impedance of the neutral line, its value has no effect on the system.

If the circuit is balanced, the equation can be written as

$$\mathbf{V}_N\left(\frac{1}{Z_0}+\frac{1}{Z_\phi}\right)=\frac{\mathbf{V}_{a'n}+\mathbf{V}_{b'n}+\mathbf{V}_{c'n}}{Z_\phi}$$

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}$$

Since the system is balanced, $\mathbf{V}_{a'n}+\mathbf{V}_{b'n}+\mathbf{V}_{c'n}=0$ and $\mathbf{V}_N=0$

This is a very important result. If $\mathbf{V}_N$=0, there is no potential difference between n and N, consequently the current in the neutral line is zero. Hence we may either remove the neutral line from a balanced Y-Y circuit ($\mathbf{I}_0$=0) or replace it with a perfect short circuit between n and N ($\mathbf{V}_N$=0). Both are convenient to use when modeling balanced 3-phase circuits.

When the system is balanced, the three line currents are

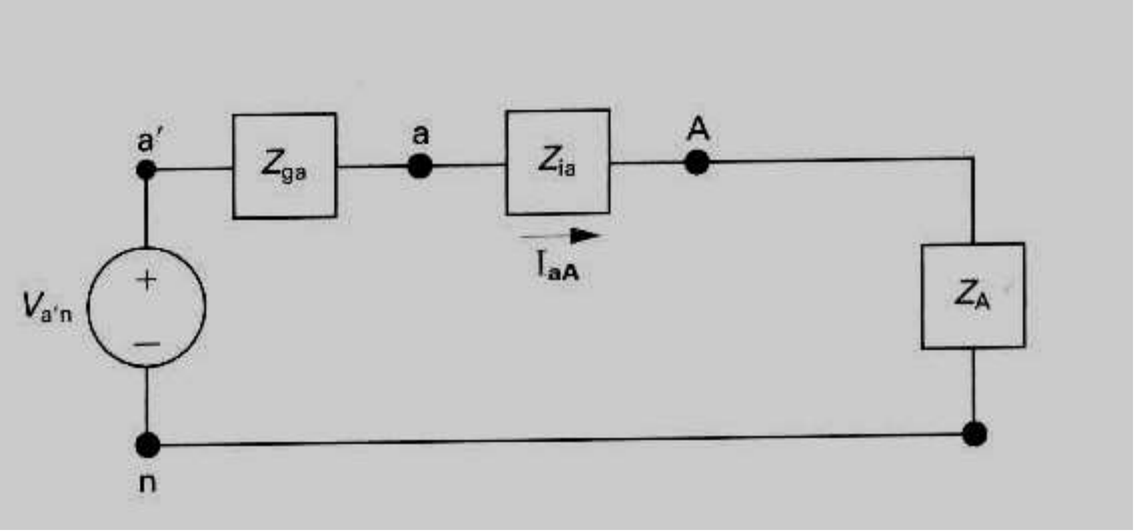
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n}}{Z_\phi}, \quad \mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n}}{Z_\phi}, \quad \mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n}}{Z_\phi}$$

The three line currents from a balanced set of three-phase currents, that is, the current in each line is equal in amplitude and frequency and is 120^0 out of phase with the other two line currents. Thus, if we calculate $\mathbf{I}_{aA}$ and know the phase sequence, we can easily write $\mathbf{I}_{bB}$ and $\mathbf{I}_{cC}$.

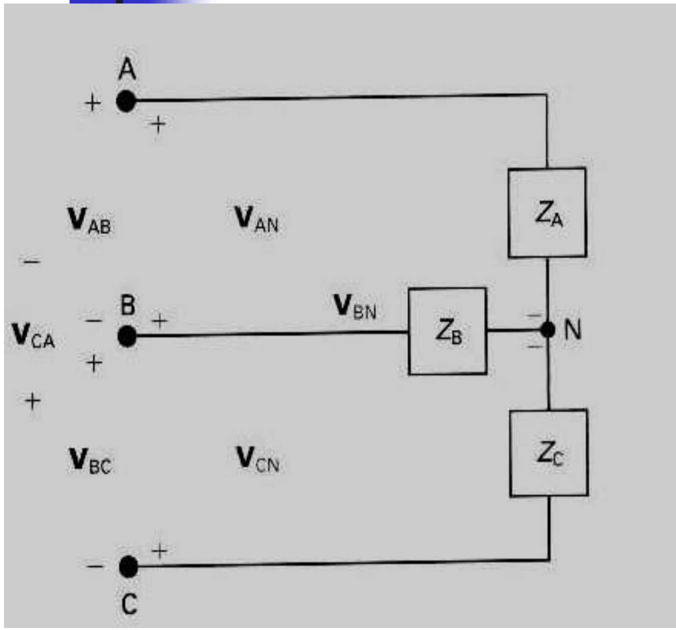
SINGLE-PHASE EQUIVALENT CIRCUIT

We can use line current equations to construct an equivalent circuit for the a-phase of the balanced Y-Y circuit. Once we solve this circuit, we can easily write the voltages and currents of the other two phases.

Caution: The current in the neutral conductor in this figure is $\mathbf{I}_{aA}$, which is not the same as the current in the neutral conductor of the balanced three-phase circuit, which is $\mathbf{I}_0=\mathbf{I}_{aA}+\mathbf{I}_{bB}+\mathbf{I}_{cC}$



LINE-TO-LINE AND LINE-TO-NEUTRAL VOLTAGES

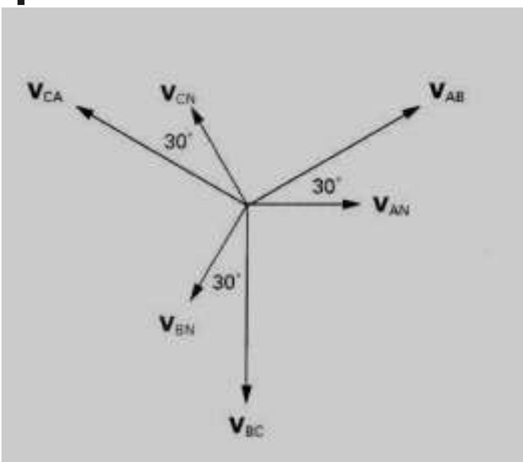


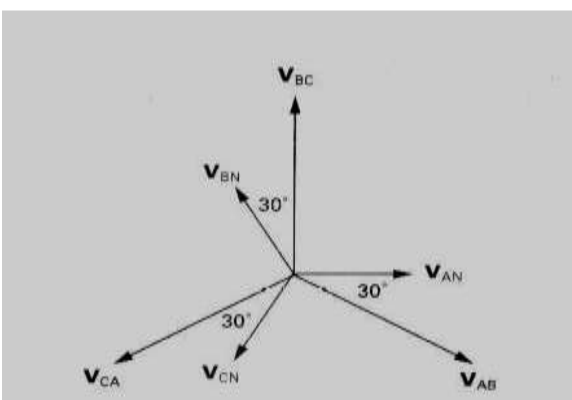
The line-to-line voltages are $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$. The line-to-neutral voltages are $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$. Using KVL

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$
$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}$$
$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}$$

For a positive (abc) phase sequence where a-phase taken as reference

$$\mathbf{V}_{AN} = V_\phi \angle 0^0 \quad \mathbf{V}_{BN} = V_\phi \angle -120^0$$
$$\mathbf{V}_{CN} = V_\phi \angle +120^0$$





$$\mathbf{V}_{AB} = V_\phi \angle 0^0 - V_\phi \angle -120^0 = \sqrt{3} V_\phi \angle 30^0$$

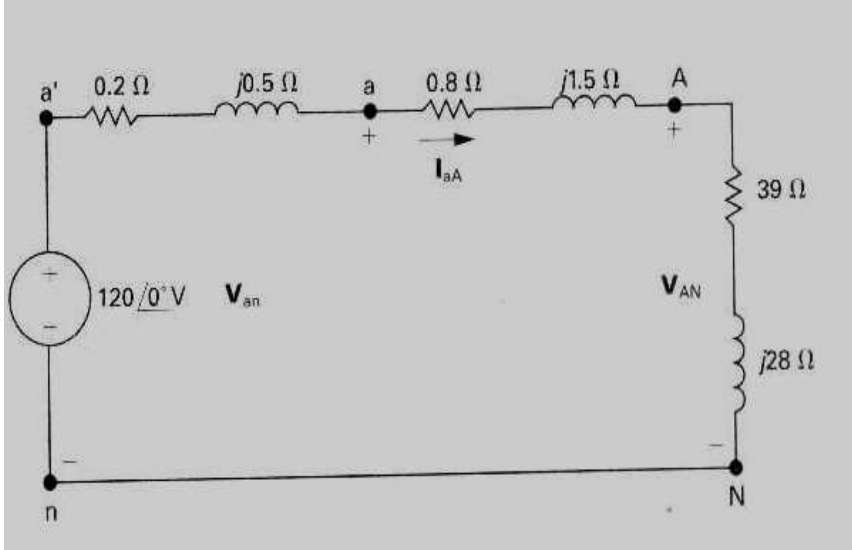
$$\mathbf{V}_{BC} = V_\phi \angle -120^0 - V_\phi \angle 120^0 = \sqrt{3} V_\phi \angle -90^0$$

$$\mathbf{V}_{AB} = V_\phi \angle 120^0 - V_\phi \angle 0^0 = \sqrt{3} V_\phi \angle 150^0$$

- The magnitude of the line-to-line voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral voltage.
- The line-to-line voltages form a balanced three-phase set of voltages.
- The set of line-to-line voltages leads the set of line-to-neutral voltages by 30^0
- For a negative sequence, the set of line-to-line voltages lags the set of line-to-neutral voltages by 30^0

EXAMPLE

A balanced three-phase Y-connected generator with positive sequence has an impedance of 0.2+j0.5 Ω/Φ and an internal voltage of 120V/ Φ. The generator feeds a balanced three-phase Y-connected load having an impedance of 39+j28 Ω/Φ. The line connecting the generator to the load is 0.8+j1.5 Ω/Φ. The a-phase internal voltage of the generator is specified as the reference phasor.



Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, $\mathbf{I}_{cC}$

$$\mathbf{I}_{aA} = \frac{120\angle 0^0}{40 + j30} = 2.4\angle -36.87^0 A$$

$$\mathbf{I}_{bB} = 2.4\angle -156.87^0 A$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^0 A$$

Calculate the three phase voltages at the load $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, $\mathbf{V}_{CN}$

$$\mathbf{V}_{AN} = (39 + j28)(2.4\angle -36.87^0) = 115.22\angle -1.19^0 V$$

$$\mathbf{V}_{BN} = 115.22\angle -121.19^0 V$$

$$\mathbf{V}_{CN} = 115.22\angle 118.81^0 V$$

Calculate the line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, $\mathbf{V}_{CA}$

$$\mathbf{V}_{AB} = (\sqrt{3}\angle 30^0)\mathbf{V}_{AN} = 199.58\angle 28.81^0 V$$

$$\mathbf{V}_{BC} = 199.58\angle -91.19^0 V$$

$$\mathbf{V}_{CA} = 199.58\angle 148.81^0 V$$

Calculate the phase voltages at the terminals of the generator $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4\angle{-36.87^0}) = 118.9\angle{-0.32^0}V$$

$$\mathbf{V}_{bn} = 118.9\angle{-120.32^0}V$$

$$\mathbf{V}_{cn} = 118.9\angle{119.68^0}V$$

Calculate the line voltages at the terminals of the generator $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$

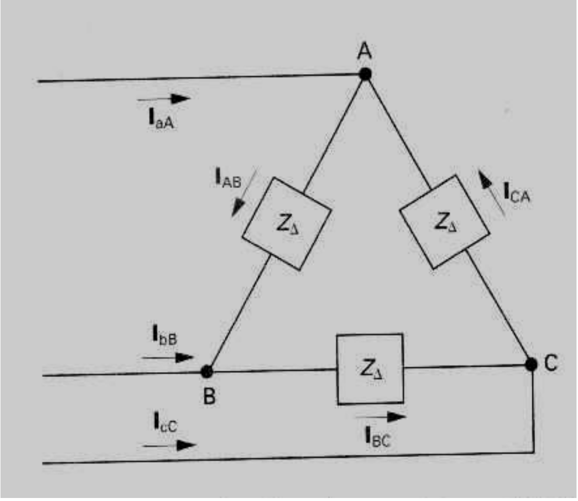
$$\mathbf{V}_{ab} = (\sqrt{3}\angle{30^0})\mathbf{V}_{an} = 205.94\angle{29.68^0}V$$

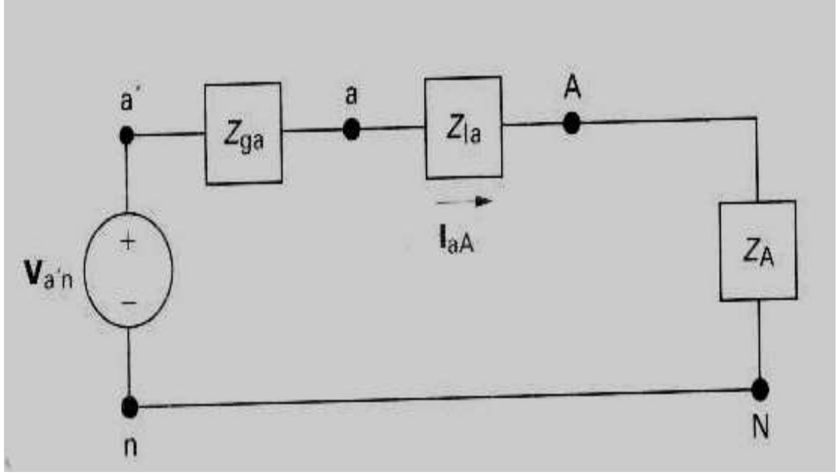
$$\mathbf{V}_{bc} = 205.94\angle{-90.32^0}V$$

$$\mathbf{V}_{ca} = 205.94\angle{149.68^0}V$$

ANALYSIS OF THE Y-Δ CIRCUIT

If the load in a three-phase circuit is connected in a delta, it can be transformed into a Y by using the Δ to Y transformation. When the load is balanced, the impedance of each leg of the Y is $Z_Y = Z_\Delta/3$. After this transformation, the a-phase can be modeled by the previous method. When a load is connected in a delta, the current in each leg of the delta is the phase current, and the voltage across each leg is the phase voltage.





To demonstrate the relationship between the phase currents and line currents, assume a positive phase sequence and

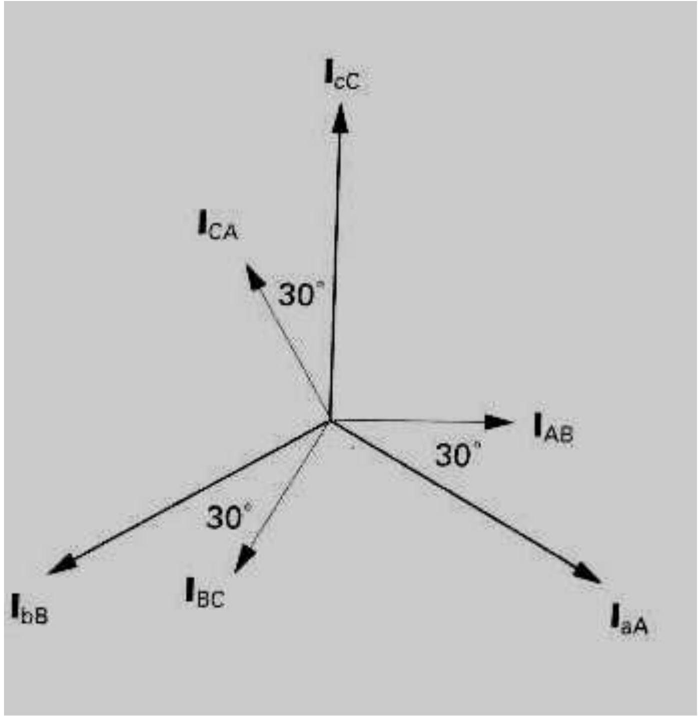
$$\mathbf{I}_{AB} = I_\phi \angle 0^0,\ \ \mathbf{I}_{BC} = I_\phi \angle -120^0,\ \ \mathbf{I}_{CA} = I_\phi \angle 120^0$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = I_\phi \angle 0^0 - I_\phi \angle 120^0 = \sqrt{3} I_\phi \angle -30^0$$

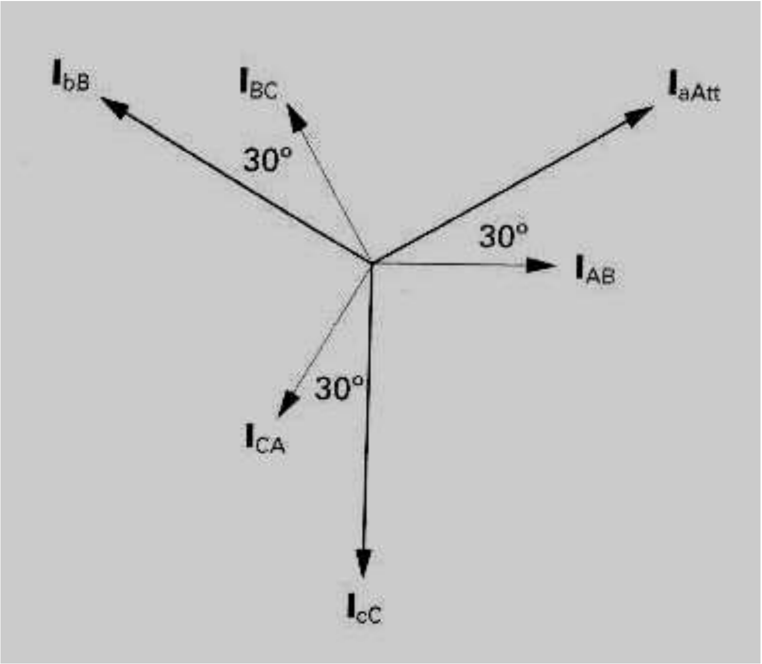
$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = I_\phi \angle -120^0 - I_\phi \angle 0^0 = \sqrt{3} I_\phi \angle -150^0$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = I_\phi \angle 120^0 - I_\phi \angle -120^0 = \sqrt{3} I_\phi \angle 90^0$$

The magnitude of the line currents is $\sqrt{3}$ times the magnitude of the phase currents and the set of line currents lags the set of phase currents by 30^0. For the negative sequence, line currents lead the phase currents by 30^0.



Positive sequence



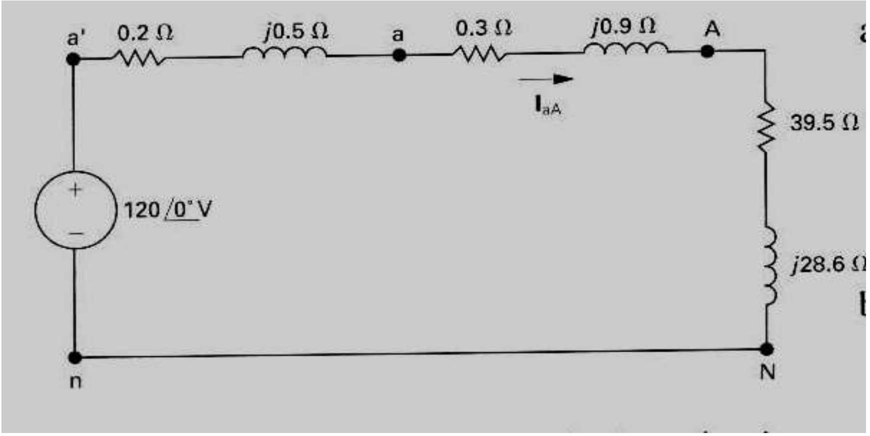
Negative sequence

EXAMPLE

The Y-connected load of the previous example feeds a Δ-connected load through a distribution line having an impedance of 0.3+j0.9Ω/Φ. The load impedance is 118.5+j85.8 Ω/Φ.

Transforming Δ load into Y and drawing a-phase of the circuit gives

$$\frac{118.5 + j85.8}{3} = 39.5 + j28.6\Omega / \phi$$



Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$

$$\mathbf{I}_{aA} = \frac{120\angle 0^0}{40 + j30} = 2.4\angle -36.87^0\, A$$

$$\mathbf{I}_{bB} = 2.4\angle -156.87^0\, A \quad \mathbf{I}_{cC} = 2.4\angle 83.13^0\, A$$

Calculate the phase voltages at the load terminals:
Because the load is Δ connected, the phase voltages are the same as the line voltages.

$$\mathbf{V}_{AN} = (39.5 + j28.6)(2.4\angle -36.87^0) = 117.04\angle -0.96^0 V$$

$$\mathbf{V}_{AB} = \sqrt{3}\angle 30^0\, \mathbf{V}_{AN} = 202.72\angle 29.04^0 V$$

$$\mathbf{V}_{BC} = 202.72\angle -90.96^0 V \quad \mathbf{V}_{CA} = 202.72\angle 149.04^0 V$$

Calculate the phase currents of the load.

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}} \angle 30^0 \mathbf{I}_{aA} = 1.39 \angle -6.87^0 \, A$$

$$\mathbf{I}_{BC} = 1.39 \angle -126.87^0 \, A \quad \mathbf{I}_{CA} = 1.39 \angle 113.13^0 \, A$$

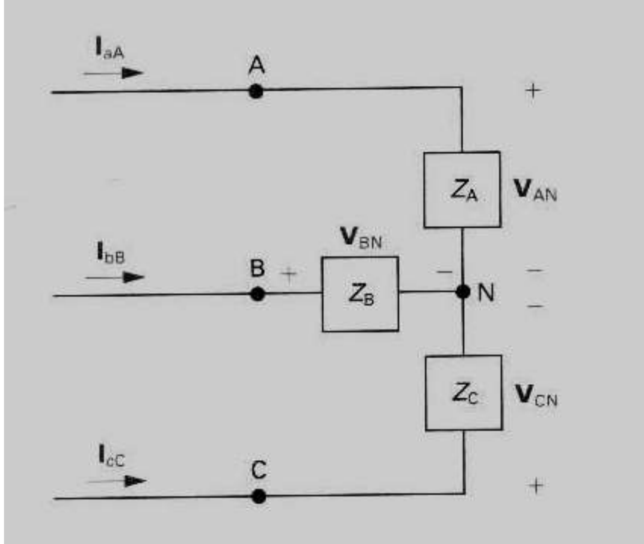
Calculate the line voltages at the source terminals

$$\mathbf{V}_{an} = (39.8 + j29.5)(2.4 \angle -36.87^0) = 118.9 \angle -0.32^0 V$$

$$\mathbf{V}_{ab} = \sqrt{3} \angle 30^0 \mathbf{V}_{an} = 205.94 \angle 29.68^0 V$$

$$\mathbf{V}_{bc} = 205.94 \angle -90.32^0 V \quad \mathbf{V}_{ca} = 205.94 \angle 149.68^0 V$$

AVERAGE POWER IN A BALANCED Y LOAD



$$P_A = |\mathbf{V}_{AN}||\mathbf{I}_{aA}|\cos(\theta_{vA} - \theta_{iA})$$

$$P_B = |\mathbf{V}_{BN}||\mathbf{I}_{bB}|\cos(\theta_{vB} - \theta_{iB})$$

$$P_C = |\mathbf{V}_{CN}||\mathbf{I}_{cC}|\cos(\theta_{vC} - \theta_{iC})$$

In a balanced three-phase system, the magnitude of each line-to-neutral voltage is the same as is the magnitude of each phase current. The argument of the cosine functions is also the same for all three phases.

$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}|$$

$$I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|$$

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

For a balanced system, the power delivered to each phase of the load is the same

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi$$

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi = \sqrt{3} V_L I_L \cos\theta_\phi$$

Where P_T is the total power delivered to the load.

COMPLEX POWER IN A BALANCED Y LOAD

Reactive power in a balanced system

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi$$

$$Q_T = 3Q_\phi = \sqrt{3} V_\phi I_\phi \sin\theta_\phi$$

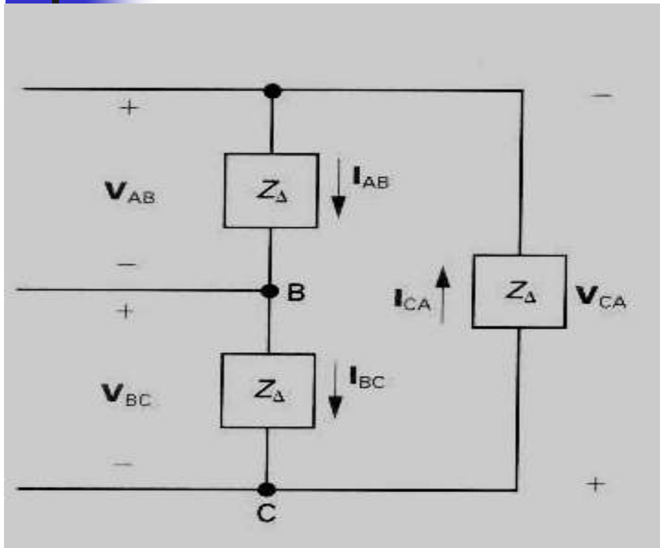
Complex power in a balanced system

$$S_\phi = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^* = \mathbf{V}_\phi \mathbf{I}_\phi^*$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*$$

$$S_T = 3S_\phi = \sqrt{3} V_L I_L \angle \theta_\phi$$

POWER CALCULATIONS IN A BALANCED DELTA LOAD



$$P_{AB} = |\mathbf{V}_{AB}||\mathbf{I}_{AB}|\cos(\theta_{vAB} - \theta_{ivAB})$$

$$P_{BC} = |\mathbf{V}_{BC}||\mathbf{I}_{BC}|\cos(\theta_{vBC} - \theta_{ivBC})$$

$$P_{CA} = |\mathbf{V}_{CA}||\mathbf{I}_{CA}|\cos(\theta_{vCA} - \theta_{ivCA})$$

For a balanced system

$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_\phi$$

$$|\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| = I_\phi$$

$$\theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA} = \theta_\phi$$

$$P_{AB} = P_{BC} = P_{CA} = P_\phi = V_\phi I_\phi \cos\theta_\phi$$

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$

$$P_T = 3\left(\frac{I_L}{\sqrt{3}}\right)V_L \cos\theta_\phi = \sqrt{3}V_L I_L \cos\theta_\phi$$

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi$$

$$Q_T = 3Q_\phi = \sqrt{3}V_\phi I_\phi \sin\theta_\phi$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*$$

$$S_T = 3S_\phi = \sqrt{3}V_L I_L \angle\theta_\phi$$

INSTANTANEOUS POWER IN THREE-PHASE CIRCUITS

In a balanced three-phase circuit, the total instantaneous power is invariant with time. Thus the torque developed at the shaft of a three-phase motor is constant, which means less vibration in machinery powered by three-phase motors.

Considering a-phase as the reference, for a positive phase sequence, the instantaneous power in each phase becomes

$$p_A = v_{AN} i_{aA} = V_m I_m \cos \omega t \cos(\omega t - \theta_\phi)$$

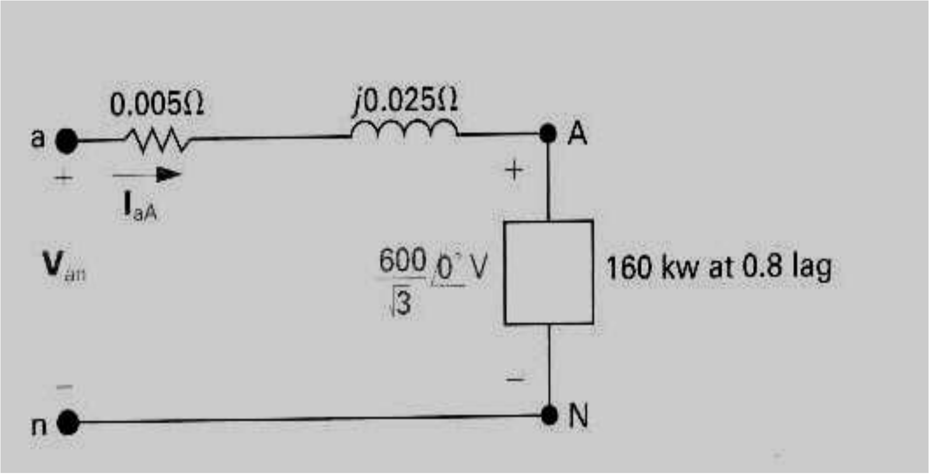
$$p_B = v_{BN} i_{bB} = V_m I_m \cos(\omega t - 120^0)\cos(\omega t - \theta_\phi - 120^0)$$

$$p_C = v_{CN} i_{cC} = V_m I_m \cos(\omega t + 120^0)\cos(\omega t - \theta_\phi + 120^0)$$

$$p_T = p_A + p_B + p_C = 1.5 V_m I_m \cos \theta_\phi$$

EXAMPLE

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The line has an impedance of 0.005+j0.025Ω/Φ. The line voltage at the terminals of the load is 600V



Single phase equivalent

Calculate the magnitude of the line current.

$$\frac{600}{\sqrt{3}}\mathbf{I}^*_{aA} = (160 + j120)10^3$$

$$\mathbf{I}^*_{aA} = 577.35\angle 36.87^0\ A$$

$$\mathbf{I}_{aA} = 577.35\angle -36.87^0\ A$$

$$I_L = 577.35A$$

Calculate the magnitude of the line voltage at the sending end of the line.

$$\mathbf{V}_{an} = \mathbf{V}_{AN} + Z_L \mathbf{I}_{aA} = \frac{600}{\sqrt{3}} + (0.005 + j0.025)(577.35 \angle -36.87^0)$$

$$= 357.51 \angle 1.57^0 V$$

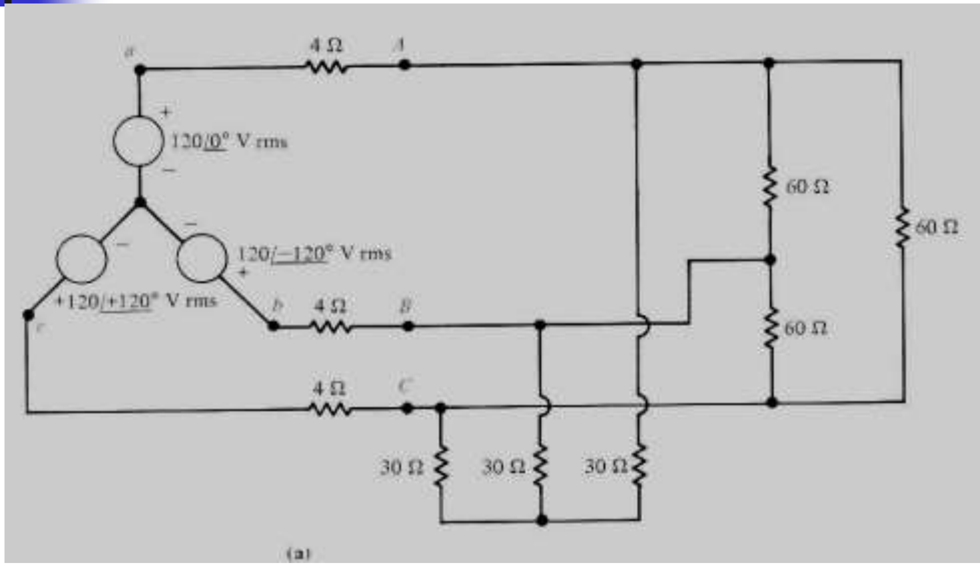
$$V_L = \sqrt{3}\left|\mathbf{V}_{an}\right| = 619.23V$$

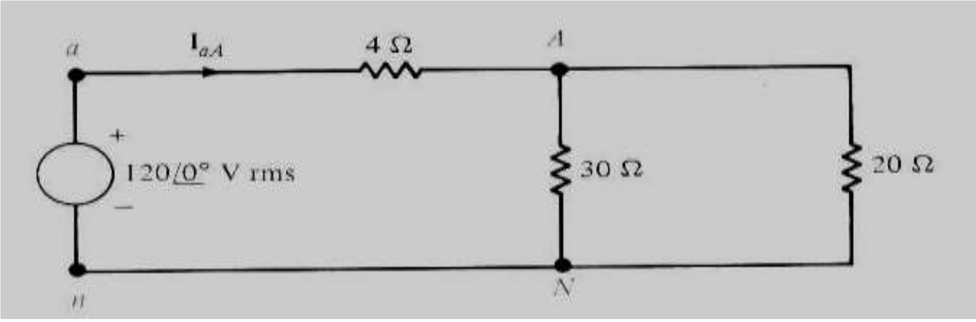
Calculate the power factor at the sending end of the line

$$\text{pf} = \cos[1.57^0 - (-36.87^0)]$$

$$= \cos 38.44^0 = 0.783 \text{ Lagging}$$

EXAMPLE





Determine all the load currents:

Transforming Δ to Y as 60/3=20Ω, and drawing a-phase of the circuit gives:

$$\mathbf{I}_{aA} = \frac{120\angle 0^0}{4+20//30} = 7.5\angle 0^0\,Arms$$

$$\mathbf{V}_{AN} = (7.5\angle 0^0)(12) = 90\angle 0^0 Vrms$$

In the original Y connected load

$$\mathbf{I}_{AN} = \frac{90\angle 0^0}{30} = 3\angle 0^0 \, Arms$$

$$\mathbf{I}_{BN} = 3\angle -120^0 \; Arms \quad \mathbf{I}_{CN} = 3\angle 120^0 \, Arms$$

For the original delta-connected load

$$\mathbf{V}_{AB} = 90\sqrt{3}\angle 0^0 + 30^0 = 155.88\angle 30^0 Vrms$$

$$\mathbf{I}_{AB} = \frac{155.88\angle 30^0}{60} = 2.6\angle 30^0 \, Arms$$

$$\mathbf{I}_{BC} = 2.6\angle -90^0 \, Arms \quad \mathbf{I}_{CA} = 2.6\angle 150^0 \, Arms$$