THE TRANSFORMER

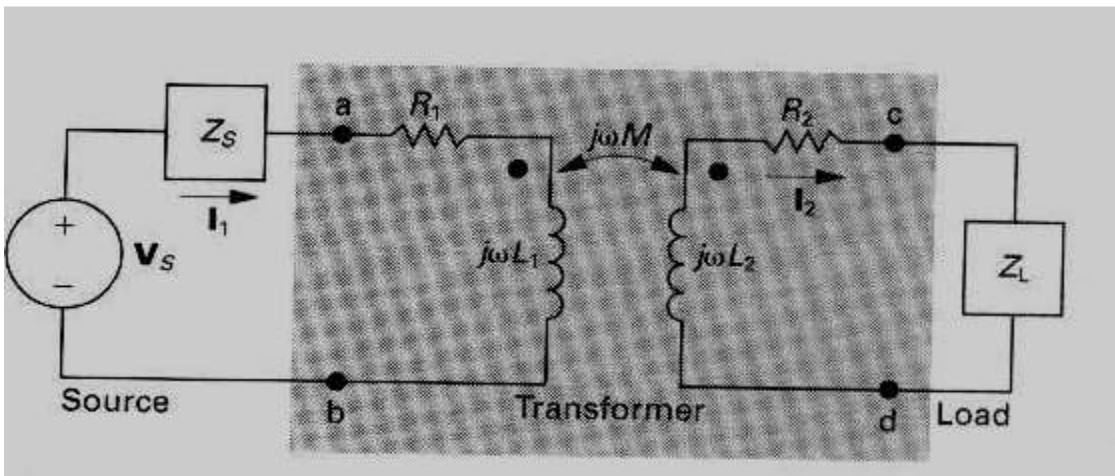
Osman Parlaktuna
Osmangazi University
Eskisehir, TURKEY

THE TRANSFORMER

- A transformer is a device that is based on magnetic coupling
- In communication, it is used to match impedances and eliminate dc signals.
- In power circuits, it is used to establish alternating current voltage levels that facilitate the transmission, distribution, and consumption of electrical power.

LINEAR TRANSFORMER

A simple transformer is formed when two coils are wound on a single core to ensure magnetic coupling.



The winding connected to the source is the **primary winding**, and the winding connected to the load is the **secondary winding**. Determine $\mathbf{I}_1$ and $\mathbf{I}_2$ as functions of circuit parameters, and also determine the input impedance at the terminals of the transformer.

$$\mathbf{V}_s = (Z_s + R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + Z_L)\mathbf{I}_2$$

$$Z_{11} = Z_s + R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\omega L_2 + Z_L$$

$$\mathbf{I}_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s = \frac{j\omega M}{Z_{22}}\mathbf{I}_1$$

$$Z_{ab} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$$

Reflected Impedance

The last term in the equation for Z_{ab} is called the reflected impedance (Z_r) because it is the equivalent impedance of the secondary coil and load impedance reflected to the primary side of the transformer.

$$Z_L = R_L + jX_L$$

$$Z_r = \frac{\omega^2 M^2}{R_2 + R_L + j(\omega L_2 + X_L)} = \frac{\omega^2 M^2 [R_2 + R_L - j(\omega L_2 + X_L)]}{(R_2 + R_L)^2 + (\omega L_2 + X_L)^2}$$

$$= \frac{\omega^2 M^2 [(R_2 + R_L) - j(\omega L_2 + X_L)]}{|Z_{22}|^2}$$

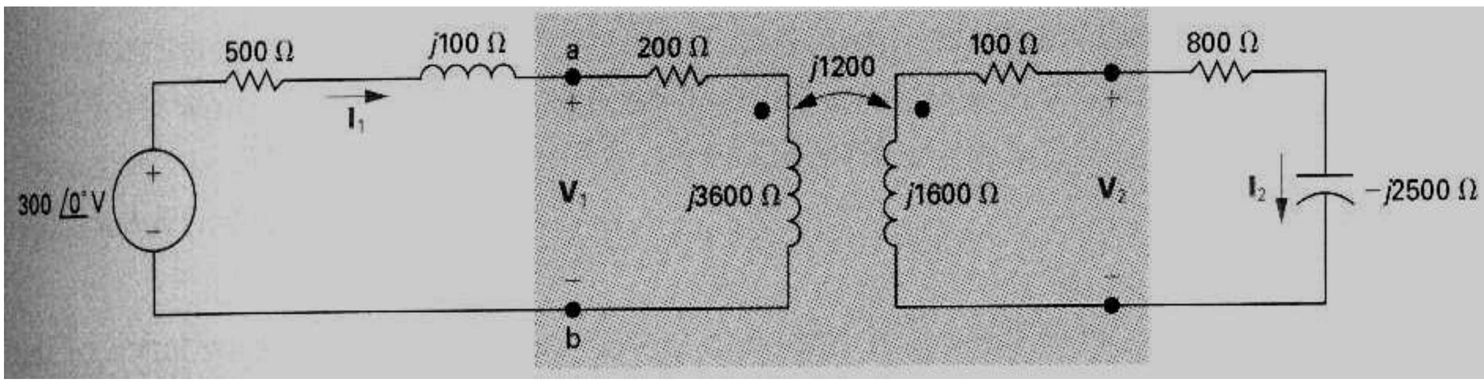
EXAMPLE

The parameters of a certain linear transformer are R_1=200Ω, R_2=100Ω L_1=9H, L_2=4H, and k=0.5. The transformer couples an impedance consisting of an 800 Ω resistor series with a 1μF capacitor to a sinusoidal voltage source. The 300 Vrms source has an internal impedance of 500+j100 Ω and a frequency of 400 rad/s.

$$j\omega L_1 = j(400)9 = j3600\Omega, \quad j\omega L_1 = j(400)4 = j1600\Omega$$

$$M = 0.5\sqrt{9\cdot 4} = 3\text{H} \Rightarrow j\omega M = j(400)3 = j1200\Omega$$

Following figure shows the frequency-domain equivalent circuit



$$Z_{11} = 500 + j100 + 200 + j3600 = 700 + j3700\Omega$$

$$Z_{22} = 100 + j1600 + 800 - j2500 = 900 - j900\Omega$$

$$Z_r = \left(\frac{1200}{|900 - j900|}\right)^2 (900 + j900) = 800 + j800\Omega$$

$$Z_{ab} = 200 + j3600 + 800 + j800 = 1000 + j4400\Omega$$

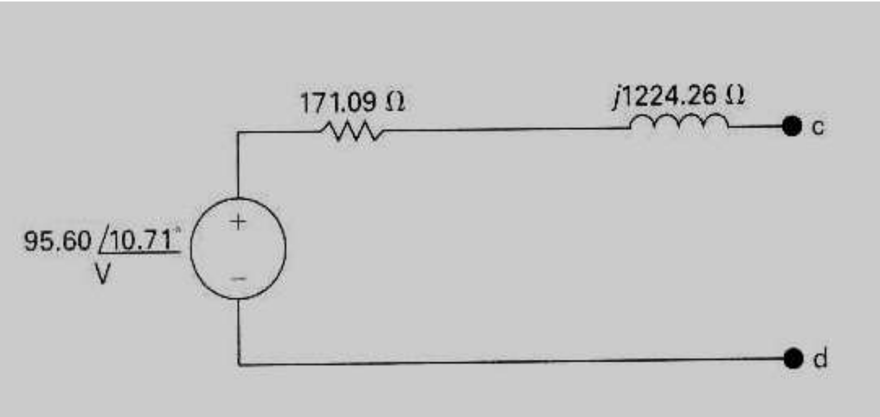
Thevenin equivalent with respect to the secondary terminals

Open circuit voltage will be j1200 times the open circuit current $\mathbf{I}_1$

$$\mathbf{I}_1 = \frac{300}{700 + j3700} = 79.67\angle{-79.29^0}\, mA$$

$$\mathbf{V}_{Th} = j1200(79.67\angle{-79.29^0}) \times 10^{-3} = 95.6\angle{10.71^0}\, V$$

$$Z_{Th} = 100 + j600 + \left(\frac{1200}{|700 + j3700|}\right)^2 (700 - j3700) = 171.09 + j1124.26\Omega$$



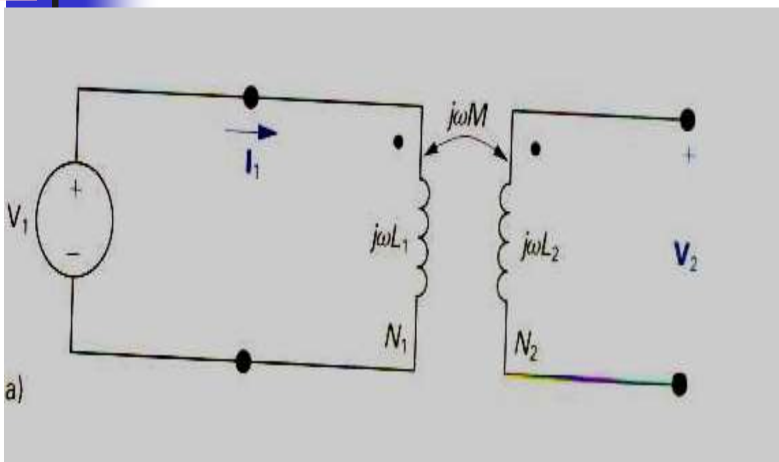
THE IDEAL TRANSFORMER

- An ideal transformer consist of two magnetically coupled coils having N_1 and N_2 turns.
- The coefficient of coupling is unity (k=1)
- The self-inductance of each coil is infinite

$$L_1 = L_2 = \infty$$

- The coil losses, due to parasitic resistance, are negligible.

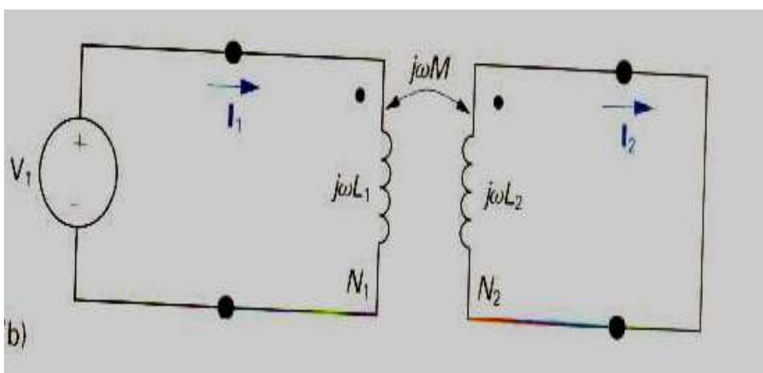
Voltage and Current Ratios



$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 \qquad \mathbf{I}_1 = \frac{\mathbf{V}_1}{j\omega L_1}$$

$$\mathbf{V}_2 = \frac{M}{L_1}\mathbf{V}_1 \qquad \text{since k} = 1, \quad \mathrm{M} = \sqrt{\mathrm{L}_1 L_2}$$

$$\mathbf{V}_2 = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = \frac{N_2}{N_1}\mathbf{V}_1$$

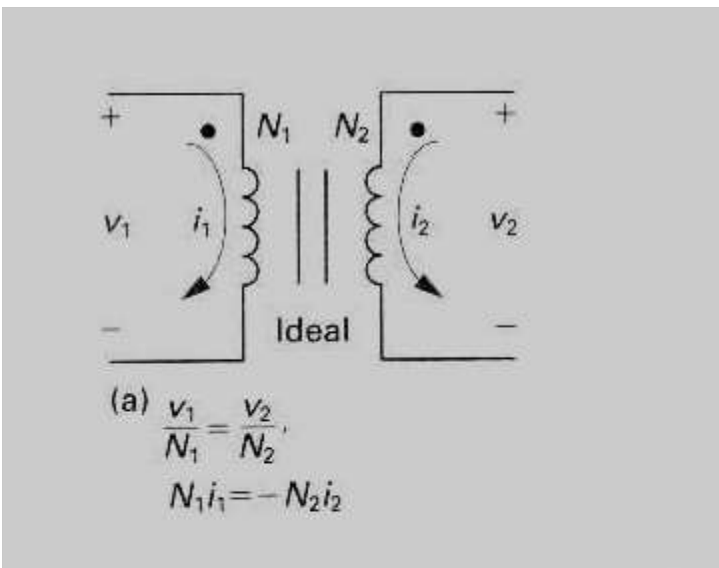


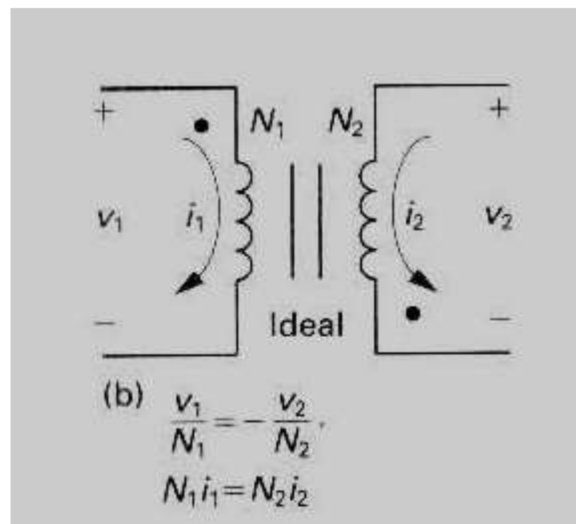
$$0 = -j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2$$

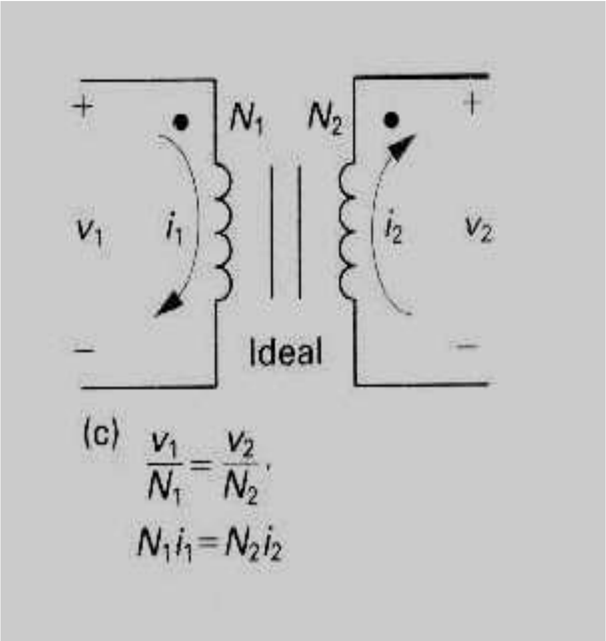
$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{L_2}{M} = \sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1}$$

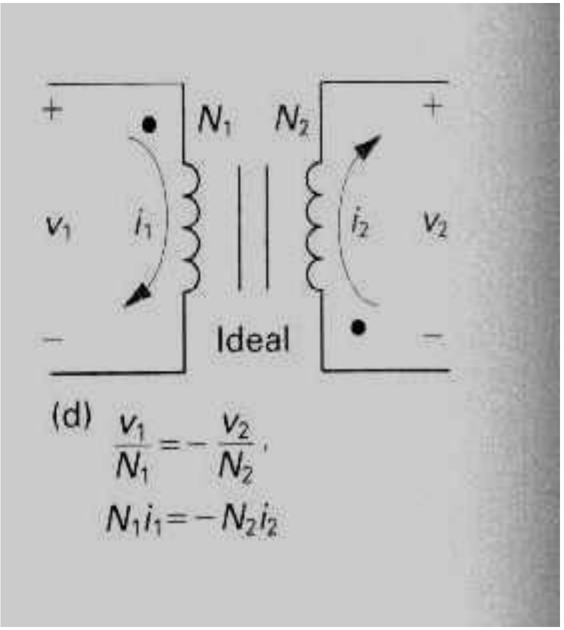
Polarity of the voltage and current ratios

1) If the coil voltages v_1 and v_2 are both positive or negative at the dot-marked terminals, use a plus sign for the voltage ratio.
2) If the coil currents i_1 and i_2 are both directed into or out of the dot-marked terminals use a minus sign for the current ratio.

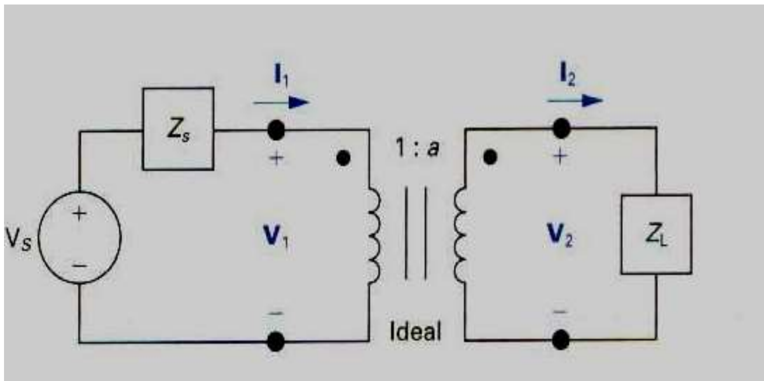





N1
N2
v1
i1
i2
v2
Ideal
(c)
$\frac{v_1}{N_1} = \frac{v_2}{N_2}$,
$N_1 i_1 = N_2 i_2$


N1
N2
v1
i1
i2
v2
Ideal
(d)
$\frac{v_1}{N_1} = -\frac{v_2}{N_2}$,
$N_1 i_1 = -N_2 i_2$

REFLECTED IMPEDANCE IN IDEAL TRANSFORMER

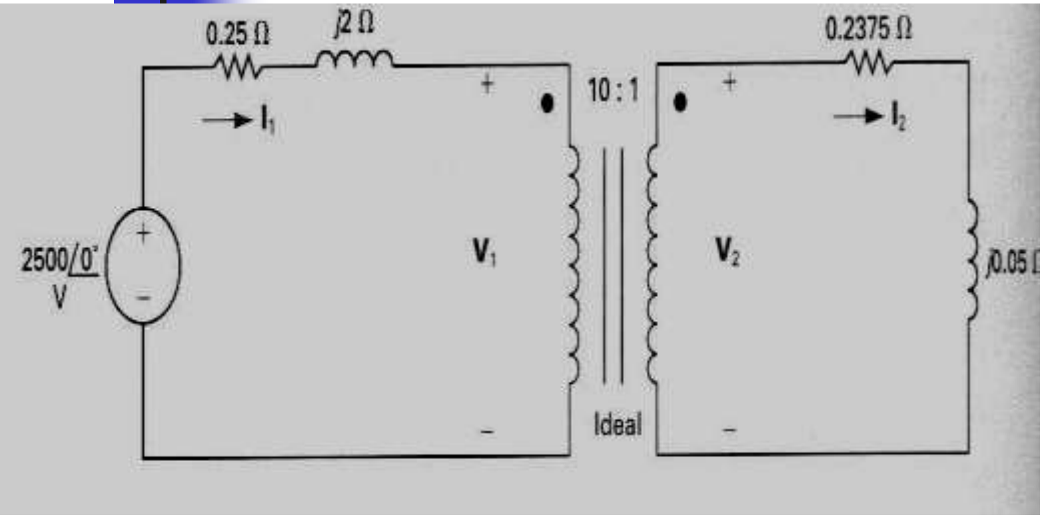


$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{a} \qquad \mathbf{I}_1 = a\,\mathbf{I}_2$$

$$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{a^2}\frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{a^2} Z_L$$

The ideal transformer's secondary coil reflects the load impedance back to the primary coil, with the scaling factor $1/a^2$.

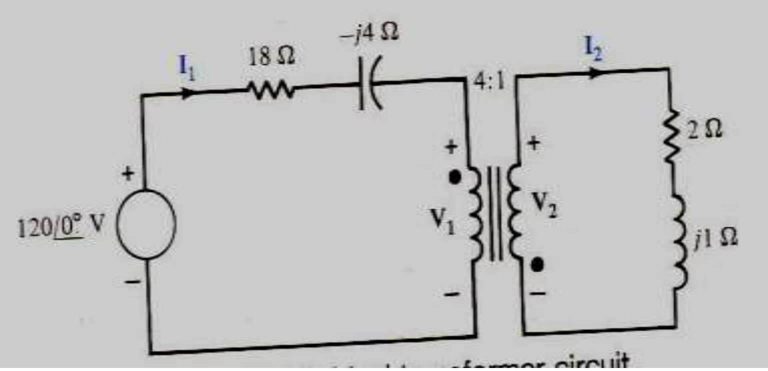
Example



Find the steady-state values for i_1, i_2, v_1, and v_2.
Use w=400 rad/s

$$2500 = (0.25 + j2)\mathbf{I}_1 + \mathbf{V}_1$$
$$\mathbf{V}_1 = 10\mathbf{V}_2 \qquad \mathbf{I}_2 = 10\mathbf{I}_1$$
$$\mathbf{V}_2 = (0.2375 + j0.05)\mathbf{I}_2$$
$$2500 = (24 + j7)\mathbf{I}_1$$
$$\mathbf{I}_1 = 100 \angle -16.26^0 A$$
$$i_1 = 100\cos(400t - 16.26^0)A$$
$$i_2 = 1000\cos(400t - 16.26^0)A$$
$$\mathbf{V}_1 = 2420 - j185 = 2427.06 \angle -4.37^0 V$$
$$v_1 = 2427.06\cos(400t - 4.37^0)V$$
$$v_2 = 242.71\cos(400t - 4.37^0)V$$

Example



Determine all indicated voltages and currents in the circuit.

$$\mathbf{V}_1 = -\frac{\mathbf{V}_2}{a}, \quad \mathbf{I}_1 = -a\mathbf{I}_2 \qquad \text{where } a = \frac{1}{4}$$

$$Z_{ref} = 16(2 + j1) = 32 + j16\Omega$$

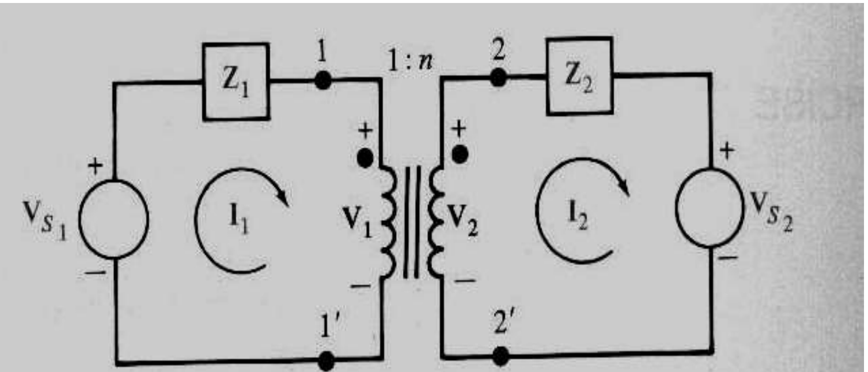
$$\mathbf{I}_1 = \frac{120}{18 - j4 + 32 + j16} = 2.33\angle{-13.5^0}\,A$$

$$\mathbf{V}_1 = \mathbf{I}_1 Z_{ref} = (2.33\angle{-13.5^0})(32 + j16) = 83.5\angle{13.07^0}\,V$$

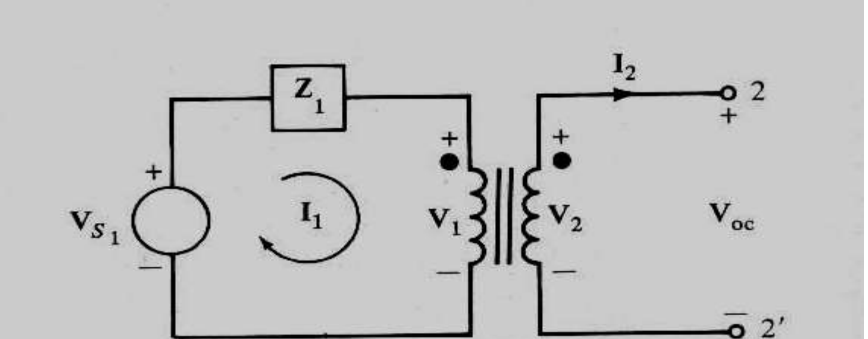
$$\mathbf{V}_2 = -a\mathbf{V}_1 = 20.88\angle{193.07^0}\,V$$

$$\mathbf{I}_2 = -\frac{\mathbf{I}_1}{a} = 9.32\angle{166.5^0}\,A$$

USING THEVENIN THEOREM IN THE ANALYSIS OF IDEAL TRANSFORMER



(a)



(b)

Determine the Thevenin Equivalent circuit that replaces the transformer and either the primary or secondary circuit.

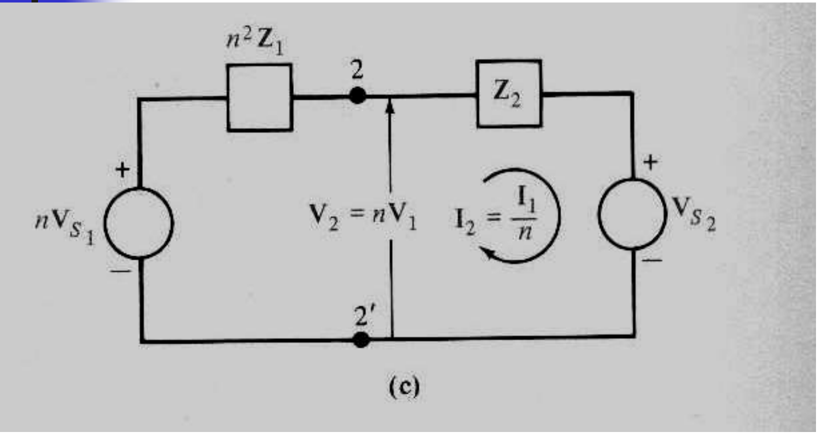
$$\mathbf{I}_1 = n\mathbf{I}_2 \qquad \mathbf{V}_1 = \frac{\mathbf{V}_2}{n}$$

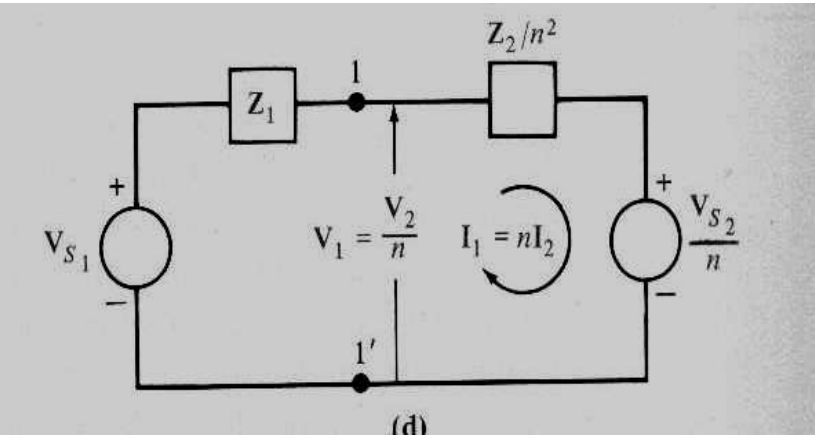
where n is the turns ratio

For open circuit voltage, $\mathbf{I}_2$=0, therefore $\mathbf{I}_1$=0. Then

$\mathbf{V}_{oc}=\mathbf{V}_2=n\mathbf{V}_1=n\mathbf{V}_{s1}$

$Z_{Th}=n^2Z_1$

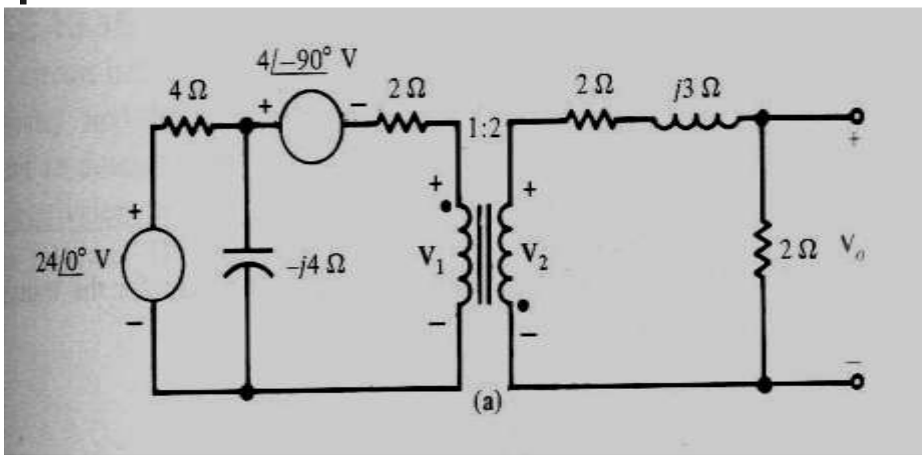


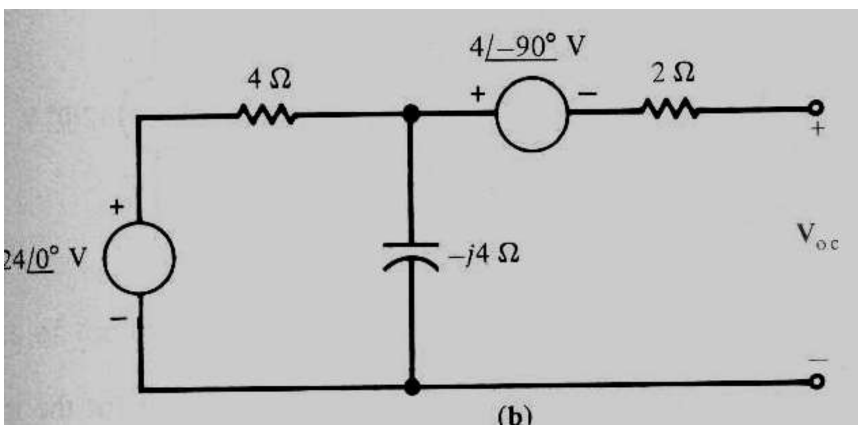


Thevenin equivalent replacing the transformer and the primary circuit. Each primary voltage is multiplied by n, each primary current is divided by n, and each primary impedance is multiplied by n^2

Thevenin equivalent replacing the transformer and the secondary circuit. Each secondary voltage is divided by n, each secondary current is multiplied by n, and each secondary impedance is divided by n^2

EXAMPLE





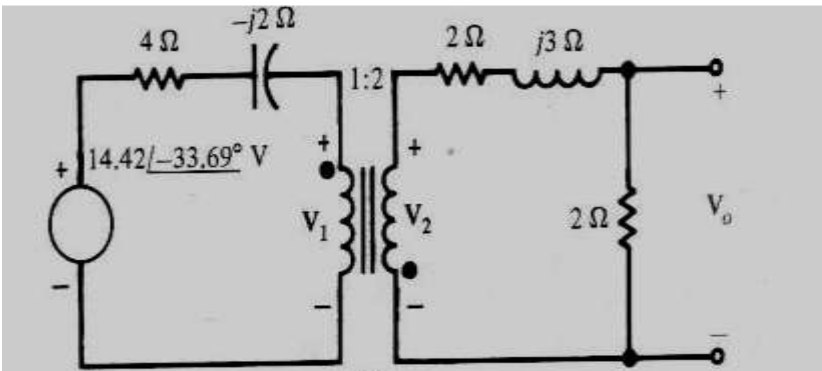
Determine $\mathbf{V}_0$

First, let us determine the Thevenin equivalent of the primary.

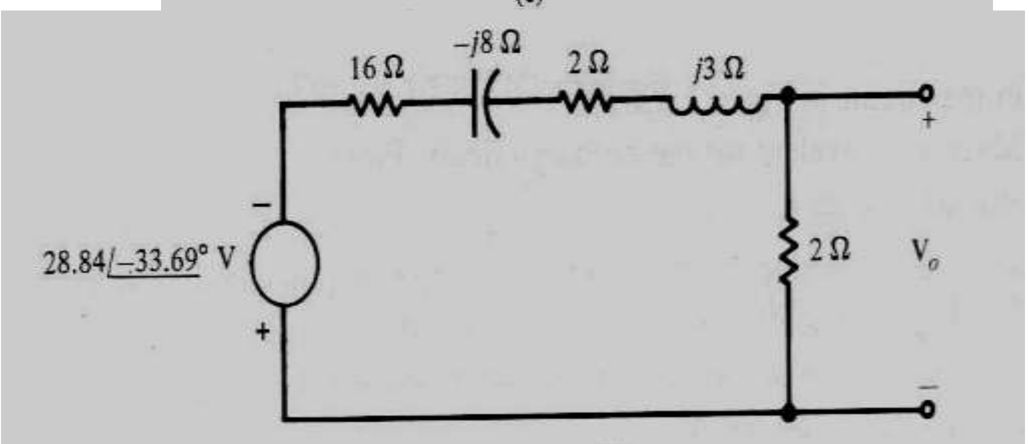
$$\mathbf{V}_{oc} = \frac{24\angle 0^0}{4 - j4}(-j4) - 4\angle -90^0$$

$$= 12 - j8 = 14.42\angle -33.69^0 V$$

$$Z_{Th} = \frac{4(-j4)}{4 - j4} + 2 = 4 - j2\Omega$$



(c)



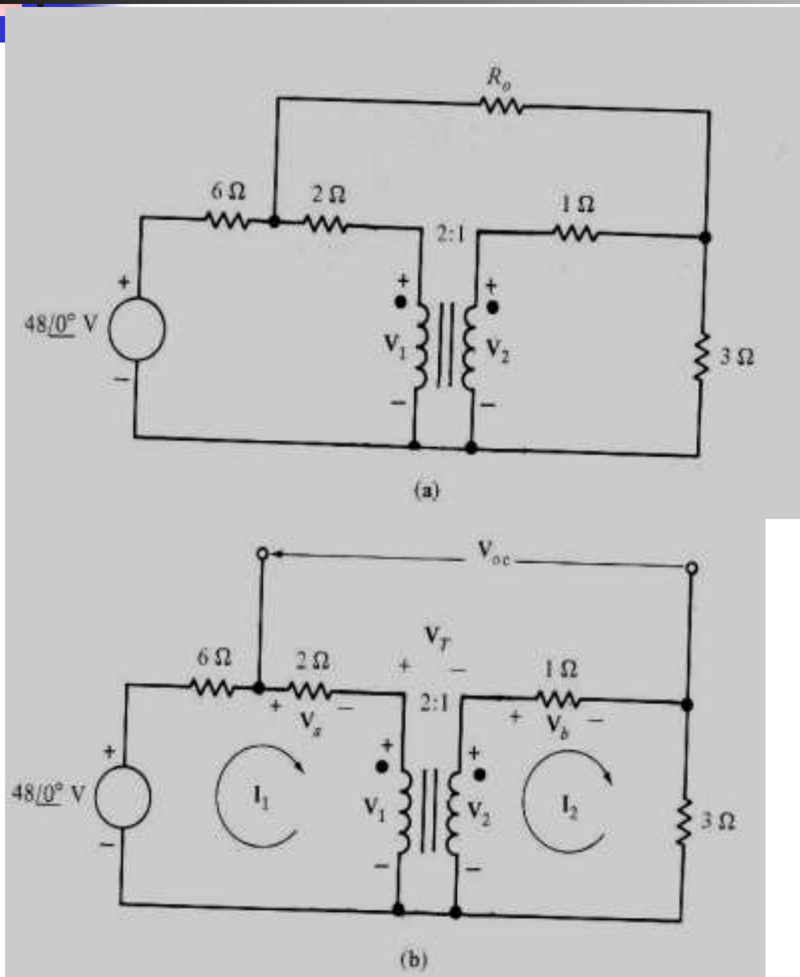
(d)

Then, reflecting the primary circuit to the secondary results in a one-mesh circuit.

$$\mathbf{V}_0 = \frac{-28.84\angle -33.69^0}{20 - j5}(2)$$

$$= 2.8\angle 160.35^0 V$$

EXAMPLE



Find the value of R_0 for maximum power transfer.

For maximum power transfer, $R_0=R_{Th}$.

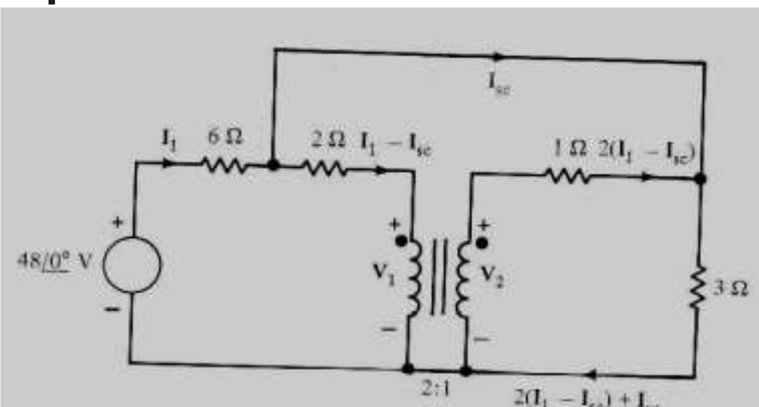
In circuit (b), secondary resistance reflected to primary as 16Ω. Thus,

$$\mathbf{I}_1 = \frac{48\angle 0^0}{6+2+16} = 2\angle 0^0 A$$

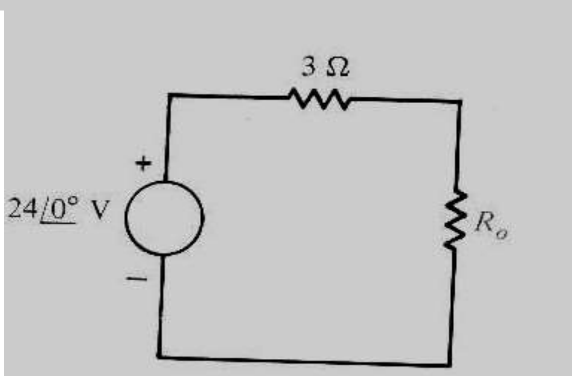
$$\mathbf{V}_1 = (2\angle 0^0)16 = 32\angle 0^0 V$$

$$\mathbf{I}_2 = 4\angle 0^0 A \quad \mathbf{V}_2 = 16\angle 0^0 V$$

$$\mathbf{V}_{oc} = \mathbf{V}_a + \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_b = 24\angle 0^0 V$$



(c)



(d)

$$48\angle 0^0 = 6\mathbf{I}_1 + 2(\mathbf{I}_1 - \mathbf{I}_{sc}) + \mathbf{V}_1$$

$$48\angle 0^0 = 6\mathbf{I}_1 + 3[2(\mathbf{I}_1 - \mathbf{I}_{sc}) + \mathbf{I}_{sc}]$$

$$\mathbf{V}_2 = (1)[2(\mathbf{I}_1 - \mathbf{I}_{sc})] + 3[2(\mathbf{I}_1 - \mathbf{I}_{sc}) + \mathbf{I}_{sc}]$$

$$\mathbf{V}_1 = 2\mathbf{V}_2$$

Solving for $\mathbf{I}_{sc}$

$$\mathbf{I}_{sc} = 8\angle 0^0\,A$$

$$Z_{Th} = \frac{24\angle 0^0}{8\angle 0^0} = 3\Omega$$

$$P_{\max} = \left[\frac{24}{2(3)}\right]^2(3) = 48W$$