ELECTROMAGNETICS -II SECOND MIDTERM EXAM

Dr. Salih FADIL

December 05, 2011

#1) A -2 mC charge starts at point $(0,1,2)$ with a velocity of $5\vec{a}_x$ m/s in magnetic field $\vec{B} = 6\vec{a}_y$ T. Determine the position of the particle after 10 s assuming the mass of the charge is 1 gr. Describe the motion of the charge

#2) A small circular loop of radius 5 cm. is centred at the origin and placed on the $z=0$ plane, if the loop carries 2 A along $\vec{a}_\phi$, calculate the magnetic flux density at $(2,2,2)$.

#3) Given the magnetic vector potential

$$\vec{A} = e^{-z}\cos y\,\vec{a}_x + (1+\sin x)\,\vec{a}_z \quad \text{Wb/m},$$

calculate the magnetic flux through a square loop described by $0 \leq x, y \leq \pi$, $z=0$. directly using $\vec{A}$.

ELECTROMAGNETICS – II SECOND MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

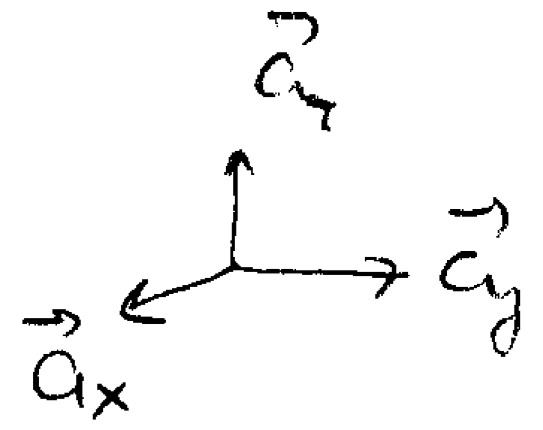
December 05, 2011

#1)

$$\vec{F} = m\vec{a} = Q(\overset{0}{\cancel{\vec{E}}} + \vec{u} \times \vec{B}) = Q(\vec{u} \times \vec{B})$$

$$\frac{du_x}{dt}\vec{a}_x + \frac{du_y}{dt}\vec{a}_y + \frac{du_z}{dt}\vec{a}_z = \frac{Q}{m}(u_x\vec{a}_x + u_y\vec{a}_y + u_z\vec{a}_z) \times 6\vec{a}_y$$

$$= \frac{-2\,10^{-3}}{1\,10^{-3}}\left[6u_x\vec{a}_z - 6u_z\vec{a}_x\right]$$



$$= -12u_x\vec{a}_z + 12u_z\vec{a}_x$$

$$\frac{du_x}{dt} = 12u_z \quad ; \quad \frac{du_z}{dt} = -12u_x$$

$$\frac{d^2u_z}{dt^2} = -12\frac{du_x}{dt} = -12(12u_z) = -144u_z$$

$$\frac{d^2u_z}{dt^2} + 144u_z = 0 \longrightarrow r^2 + 144 = 0 \qquad \begin{array}{l} r_1 = j12 \\ r_2 = -j12 \end{array}$$

$$u_z(t) = \hat{A}_1 e^{j12t} + \hat{A}_2 e^{-j12t} \longrightarrow \boxed{u_z(t) = A_1\cos(12t) + A_2\sin(12t)}$$

$$u_x = -\frac{1}{12}\frac{du_z}{dt} = -\frac{1}{\cancel{12}}\left[-\cancel{12}A_1\sin(12t) + \cancel{12}A_2\cos(12t)\right]$$

$$\boxed{u_x(t) = +A_1\sin(12t) - A_2\cos(12t)}$$

$$\frac{du_y}{dt} = 0 \rightarrow \boxed{u_y(t) = A_3}$$

$u_x(0) = 5, \quad u_y(0) = 0 \qquad u_z(0) = 0$

$\boxed{+5 = -A_2} \qquad \boxed{A_1 = 0} \qquad \boxed{A_3 = 0}$

$u_x(t) = 5\cos(12t)$ m/s.

$u_y(t) = 0$

$u_z(t) = -5\sin(12t)$ m/s.

$$\frac{dx}{dt} = 5\cos(12t) \rightarrow x(t) = \frac{5}{12}\sin(12t) + K_1$$

$$\frac{dy}{dt} = 0 \longrightarrow y(t) = K_2$$

$$\frac{dz}{dt} = -5\sin(12t) \longrightarrow z(t) = +\frac{5}{12}\cos(12t) + K_3$$

$x(0) = 0 = 0 + K_1$; $\boxed{K_1 = 0}$

$y(0) = 1 = K_2 \qquad \boxed{K_2 = 1}$

$z(0) = 2 = +\frac{5}{12} + K_3 \qquad K_3 = 2 - \frac{5}{12} = \frac{\overbrace{19}^{24-5}}{12} = \frac{19}{12} \qquad \boxed{K_3 = \frac{19}{12}}$

$$x(t) = \frac{5}{12}\sin(\overset{rad}{12t})$$

$$y(t) = 1$$

$$z(t) = +\frac{5}{12}\cos(\overset{rad}{12t}) + \frac{19}{12}$$

$x(10) = \frac{5}{12}\sin(\overset{rad.}{120})$

$120 = 19 \times 2\pi + \alpha$

$\alpha = 120 - 19 \times 2\pi = 0{,}61948$ rad.

$x(10) = 0{,}24192$

$y(10) = 1$

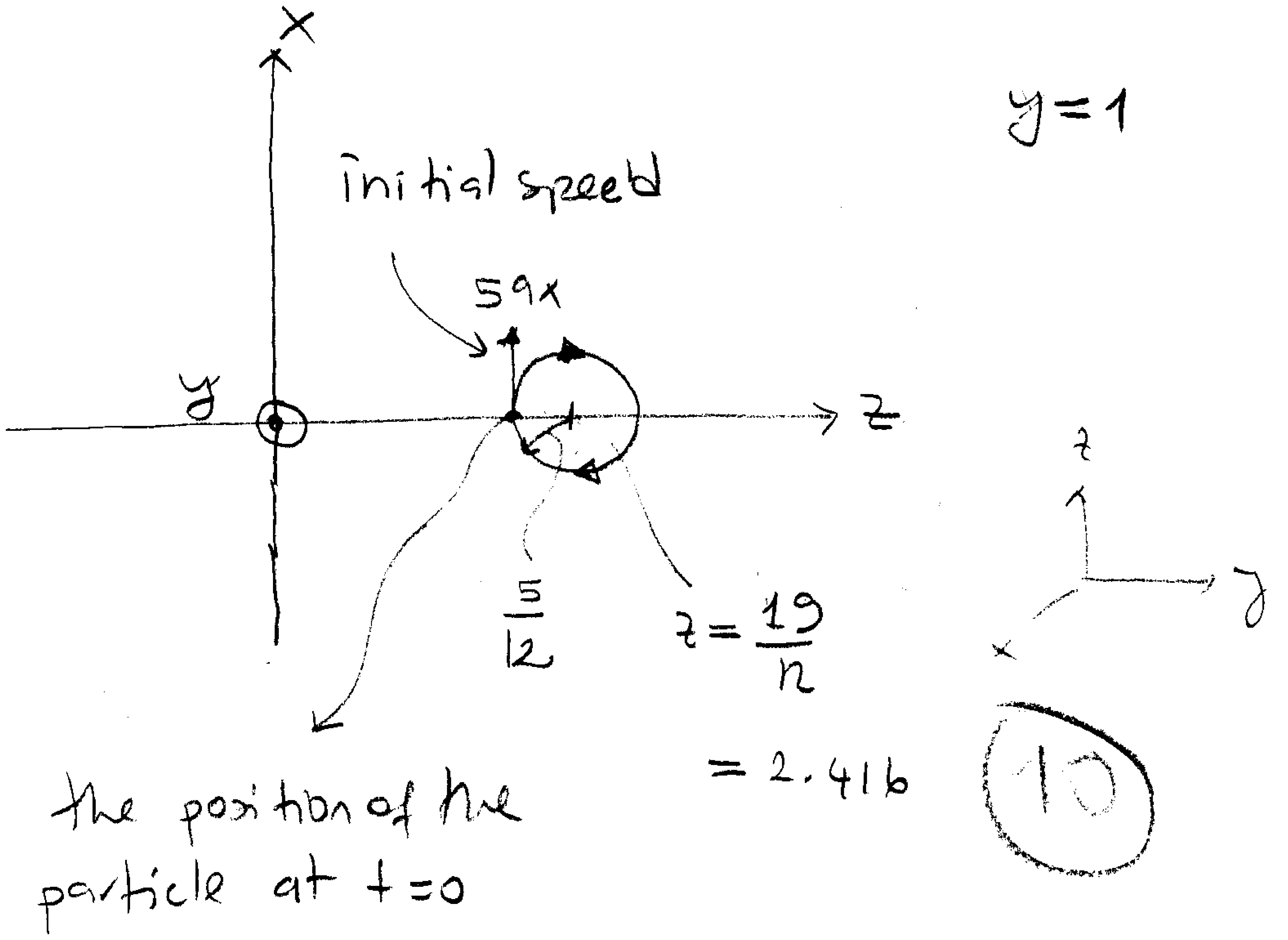
$z(10) = 1.922$

$x(t) = \frac{5}{12}\sin(12t)$

$z(t) - \frac{19}{12} = +\frac{5}{12}\cos(12t)$

$$x^2 + \left(z - \frac{19}{12}\right) = \left(\frac{5}{12}\right)^2 \underbrace{\left(\sin^2(12t) + \cos^2(12t)\right)}_{1}$$

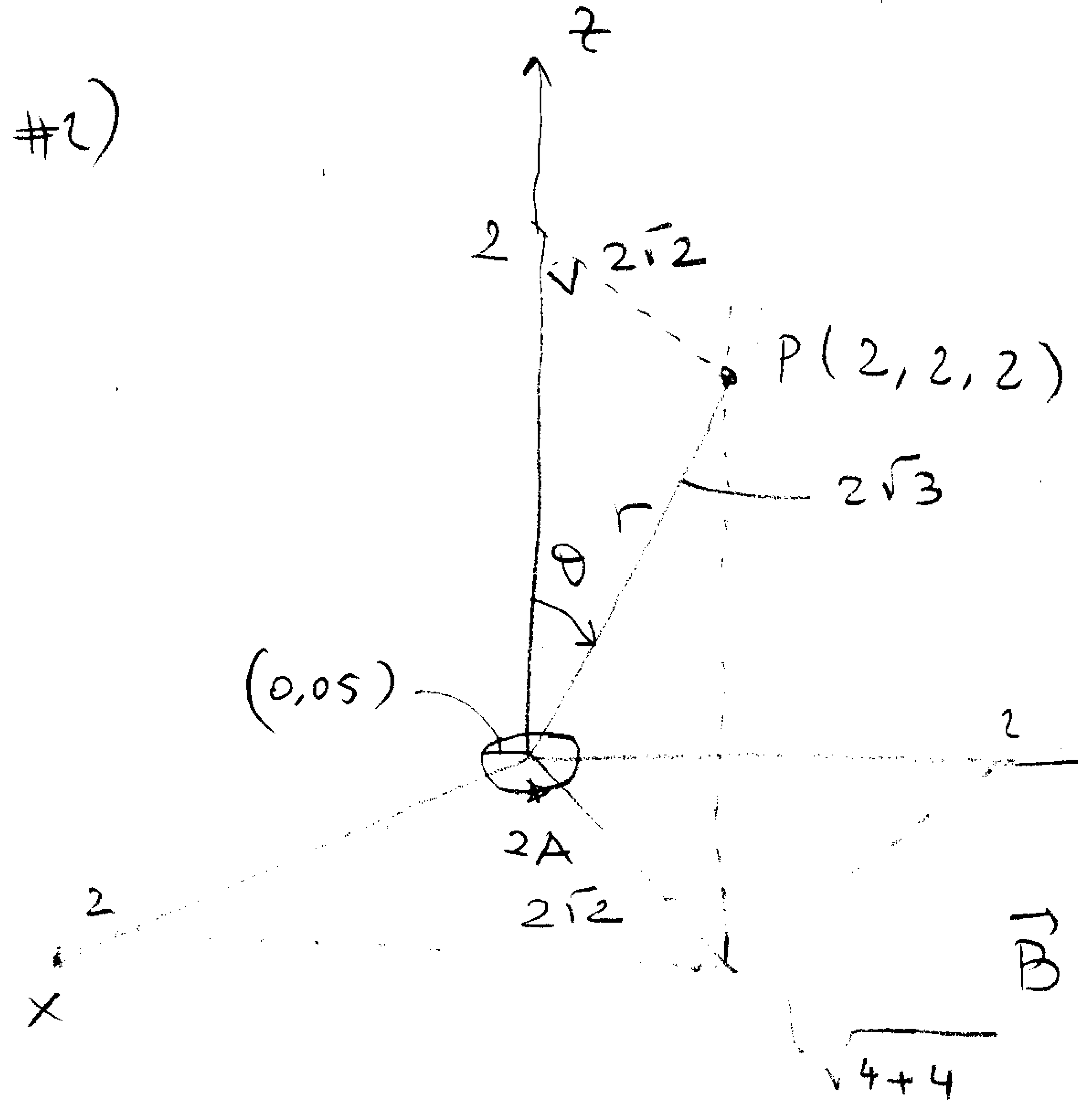
$$x^2 + \left(z - \frac{19}{12}\right) = \left(\frac{5}{12}\right)^2$$



10

The particle moves on the circular path whose center and radius are $(0, 1, \frac{19}{12})$ and $(\frac{5}{12})$, respectively in the direction shown in the figure.

#2)



$\vec{B}_p = ?$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\, \vec{a}_r + \sin\theta\, \vec{a}_\theta\right)$$

$m = I a^2 \pi = 2\,(0{,}05)^2 \pi = 5\pi\, 10^{-3}$ A.m

$r = (4+4+4)^{1/2} = 2\sqrt{3}$ m.

$\cos\theta = \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$, $\sin\theta = \frac{\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{3}} = \sqrt{2/3}$

$$\vec{B} = \frac{\cancel{4\pi}\, 10^{-7}\, 5\pi\, 10^{-3}}{\cancel{4\pi}\, (12)^{3/2}} \left(2\,\frac{1}{\sqrt{3}}\,\vec{a}_r + \sqrt{\tfrac{2}{3}}\,\vec{a}_\theta \right) \text{ T}$$

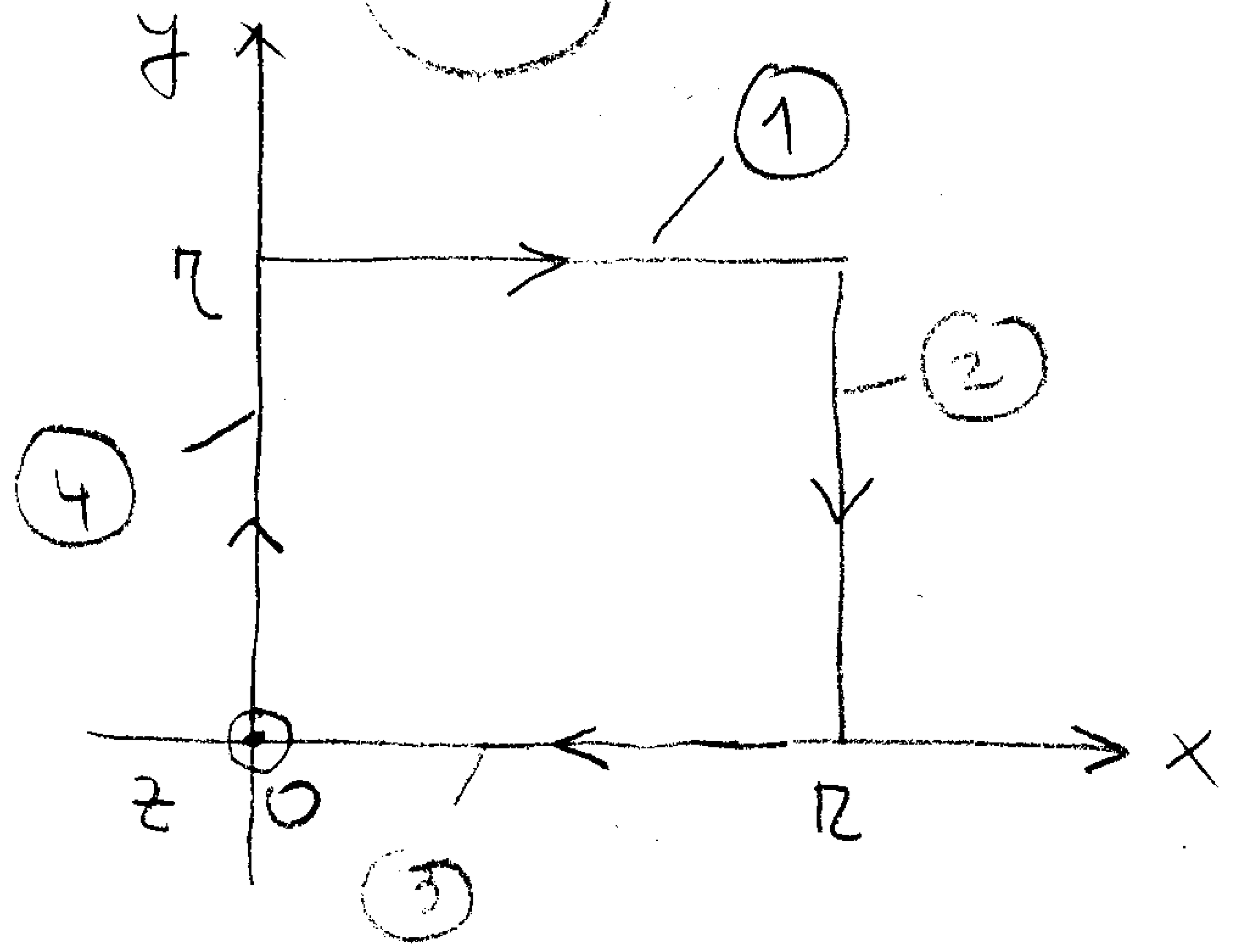
$$\vec{B} = \frac{5\pi}{(12)^{3/2}}\, 10^{-10}\, \frac{2}{\sqrt{3}}\, \vec{a}_r + \frac{5\pi}{(12)^{3/2}} \cdot 10^{-10} \sqrt{\tfrac{2}{3}}\, \vec{a}_\theta$$

$$\vec{B} = \left(0,4363\, \vec{a}_r + 0,3085\, \vec{a}_\theta \right) 10^{-10} \text{ T}$$

$$\vec{B} = 0,04363\, \vec{a}_r + 0,03085\, \vec{a}_\theta \text{ nT}$$

30

#3) 30



$$\psi = \oint_L \vec{A} \cdot \vec{dl}$$

$$\vec{A} = e^{-z} \cos y\, \vec{a}_x + (1 + \sin x)\, \vec{a}_z \text{ Wb/m}$$

$$\psi = \left(\int_{①} + \int_{②} + \int_{③} + \int_{④} \right) \vec{A} \cdot \vec{dl}$$

<u>segment - 1</u>

$\vec{dl} = dx\, \vec{a}_x$, $y = \pi$, $z = 0$

$$\int_{①} e^{-z} \cos y\, dx = e^{0} \cos(\pi) \int_{x=0}^{\pi} dx = -1\,(\pi - 0) = -\pi$$

<u>segment - 2</u>

$\vec{dl} = dy\, \vec{a}_y$; $x = \pi$, $z = 0$

$$\int_{②} \vec{A} \cdot \vec{dl} = 0$$

segment - 4

$$\int_{④} \vec{A} \cdot \vec{dl} = 0$$

segment - 3

$\vec{dl} = dx\, \vec{a_x}$ $y=0$, $z=0$

$$\int_{③} \vec{A} \cdot \vec{dl} = e^0 \cos(0) \int_{\pi}^{0} dx = 1 \; x \Big|_{\pi}^{0} = (0-\pi) = -\pi$$

$$\psi = -\pi + 0 + (-\pi) + 0 = -2\pi.$$