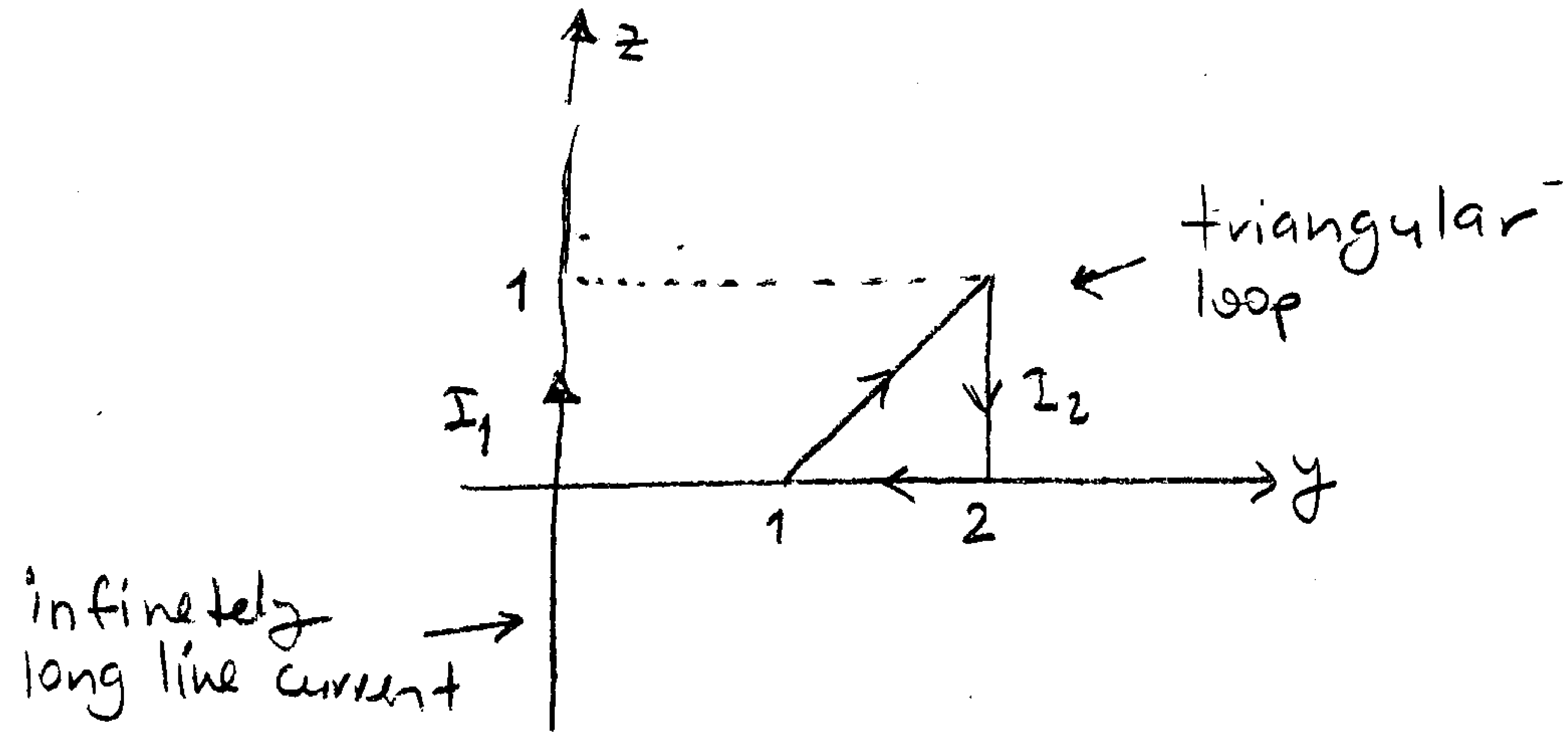
ELECTROMAGNETICS II SECOND MIDTERM EXAM

Dr. Salih FADIL May 08, 2014

#1) Calculate the mutual inductance between the infinite line current and the triangular loop shown in the following figure.



#2) Line $x=1, y=1$ carries a filamentary current $50\pi\ A$ along $\vec{a}_z$ while the $y=5$ plane carries $20\ mA/m$ along $\vec{a}_z$, Find $\vec{H}$ at the origin.

#3) Region 1 $(y \le 0)$ consists of magnetic material for which $\mu_{r1}=2$ while region 2 $(y \ge 0)$ is free space. If $\vec{B}_1 = 40\vec{a}_x + 50\vec{a}_y - 30\vec{a}_z\ mWb/m^2$, calculate

a) $\vec{B}_2$

b) $\dfrac{H_1}{H_2}$

c) $\dfrac{\tan\theta_1}{\tan\theta_2}$

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SOLUTION MANUAL

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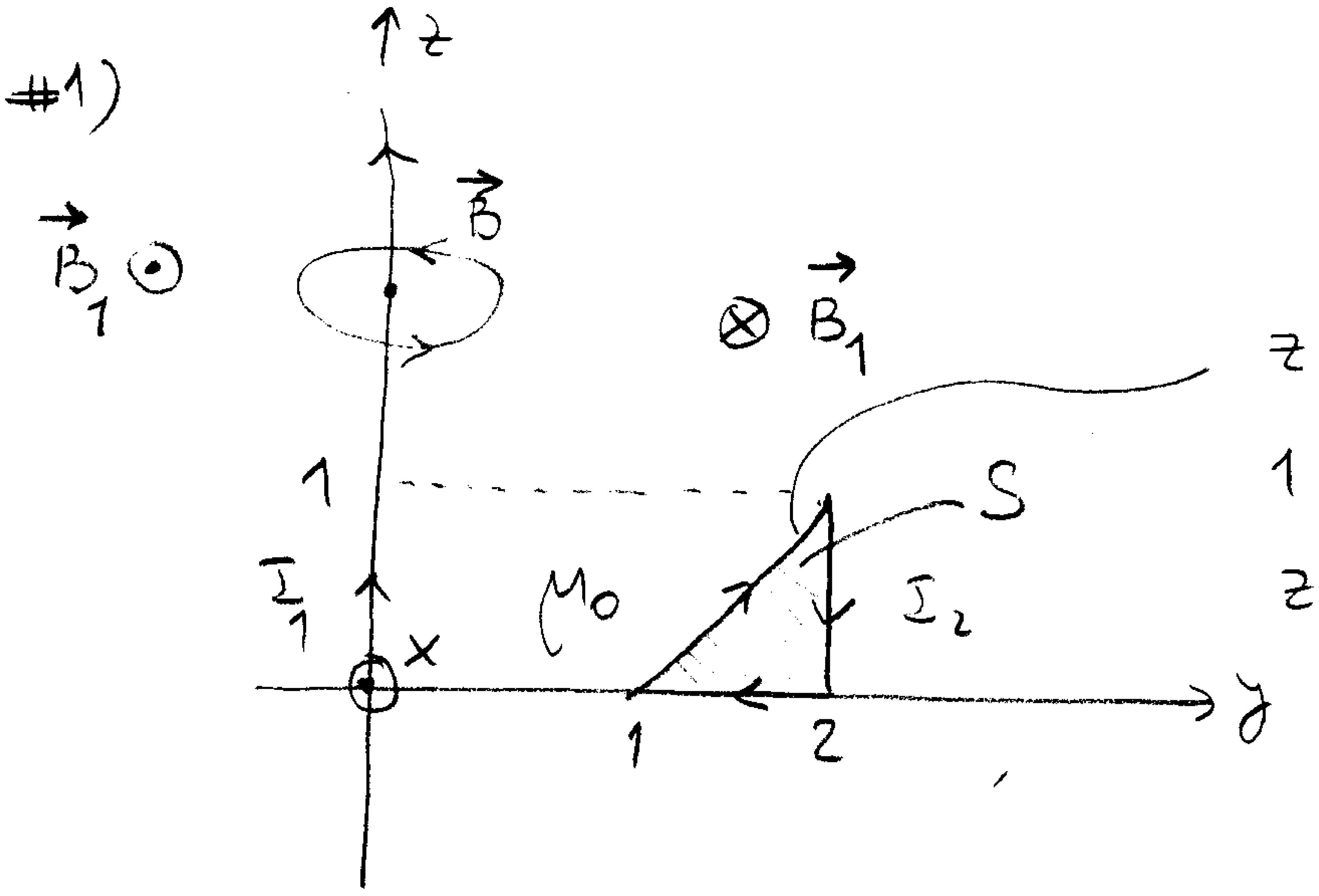
#1)



$z = my + n \qquad m = 1$

$1 = 2 + n \qquad n = -1$

$z = y - 1$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho}\vec{a}_\phi = \frac{\mu_0 I_1}{2\pi y}(-\vec{a}_x) \qquad \vec{dS} = dy\,dz\,(-\vec{a}_x)$$

$$\lambda_{21} = \Psi_{21} = \int_S \vec{B}_1 \cdot \vec{dS} = \int_{y=1}^{2}\int_{z=0}^{y-1} \frac{\mu_0 I_1}{2\pi y}\,dy\,dz$$

$$= \frac{I_1 \mu_0}{2\pi}\int_{y=1}^{2} \frac{1}{y}\left(z\Big|_0^{y-1}\right)dy = \frac{I_1\mu_0}{2\pi}\int_{y=1}^{2}\left(\frac{y-1}{y}\right)dy$$

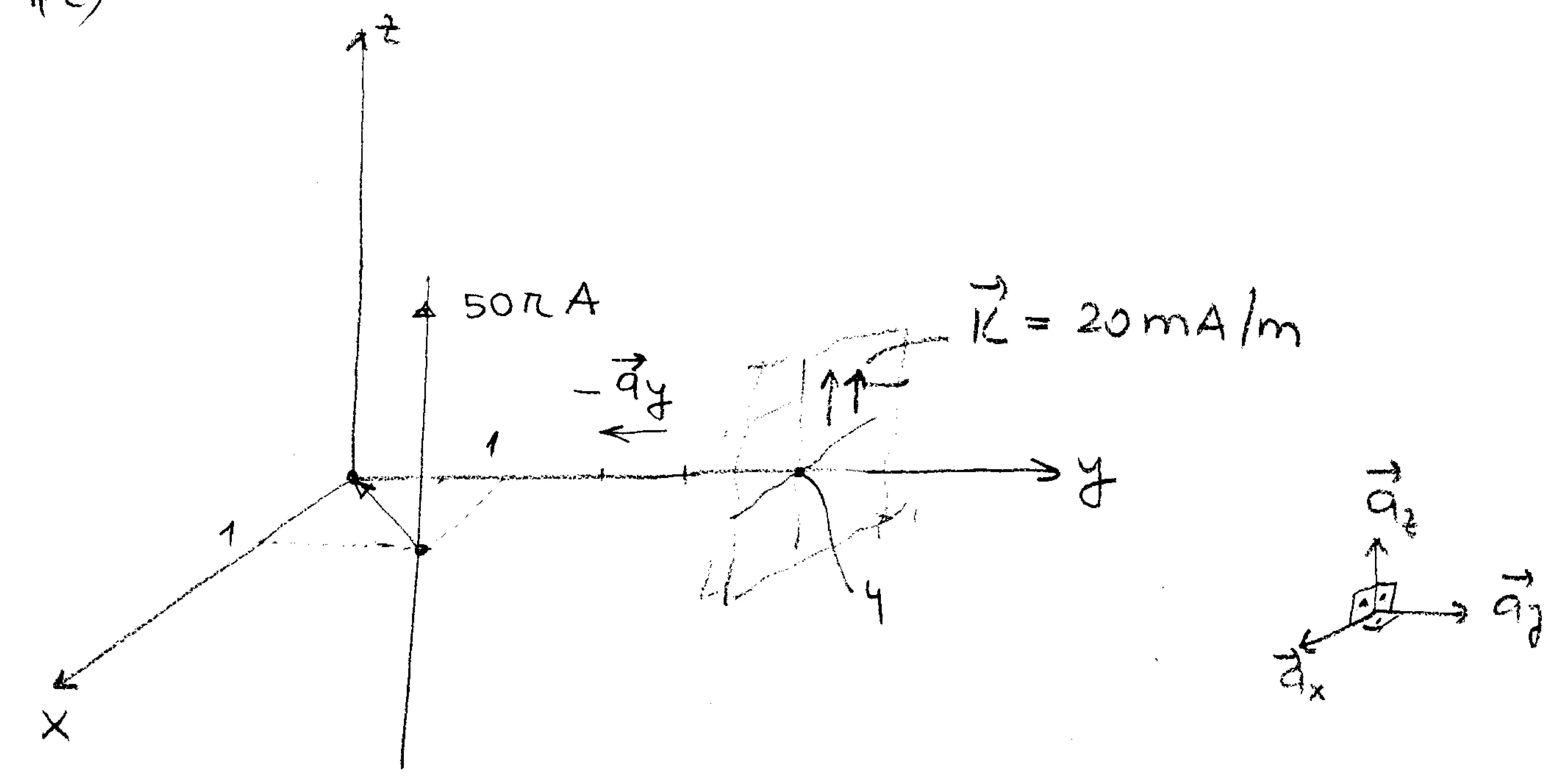
$$= \frac{I_1\mu_0}{2\pi}\int_1^2\left(1-\frac{1}{y}\right)dy = \frac{I_1}{2\pi}\left\{ y\Big|_1^2 - \ln y\Big|_1^2 \right\}\mu_0$$

$$\lambda_{21} = \frac{I_1\mu_0}{2\pi}\left\{(2-1) - (\ln 2 - \cancel{\ln 1}^{\,0})\right\} = \frac{I_1\mu_0}{2\pi}(1-\ln 2)$$

$$M_{21} = M = \frac{\lambda_{21}}{I_1} = \frac{(1-\ln 2)\mu_0}{2\pi} = \frac{(1-\ln 2)\,\cancel{4\pi}^{\,2}\,10^{-7}}{\cancel{2\pi}} = 0{,}613\ 10^{-7}\ H$$

$$= 61{,}3\ nH$$

#2)



$$\vec{H}_o = \vec{H}_{o,L} + \vec{H}_{o,S} \qquad \vec{H}_{o,L} = \frac{I}{2\pi\rho}\,\vec{a}_\phi$$

$$\vec{a}_\phi = \vec{a}_\ell \times \vec{a}_\rho \;, \quad \vec{\rho} = (0,0)-(1,1) = -\vec{a}_x - \vec{a}_y \;, \quad \vec{a}_\rho = \frac{1}{\sqrt{2}}(-\vec{a}_x - \vec{a}_y)$$

$$\rho = \sqrt{2} \qquad \vec{a}_\ell = \vec{a}_z$$

$$\vec{a}_\phi = \vec{a}_z \times \left(\frac{-\vec{a}_x - \vec{a}_y}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(-\vec{a}_y + \vec{a}_x)$$

$$\vec{H}_{o,L} = \frac{I}{2\pi\sqrt{2}}\,\frac{1}{\sqrt{2}}(-\vec{a}_y + \vec{a}_x) = \frac{I}{4\pi}(-\vec{a}_y + \vec{a}_x) = \frac{50\cancel{\pi}}{4\cancel{\pi}}(-\vec{a}_y + \vec{a}_x)$$

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$$= 12{,}5\,(-\vec{a}_y + \vec{a}_x) \;\; A/m$$

$$\vec{H}_{o,S} = \frac{1}{2}\vec{K} \times \vec{a}_n = \frac{1}{\cancel{2}}\,\cancel{20}\,10^{-3}\,\vec{a}_z \times (-\vec{a}_y) = 10\,10^{-3}\,\vec{a}_x$$

$$= 10^{-2}\,\vec{a}_x = 0{,}01\,\vec{a}_x \;\; A/m$$

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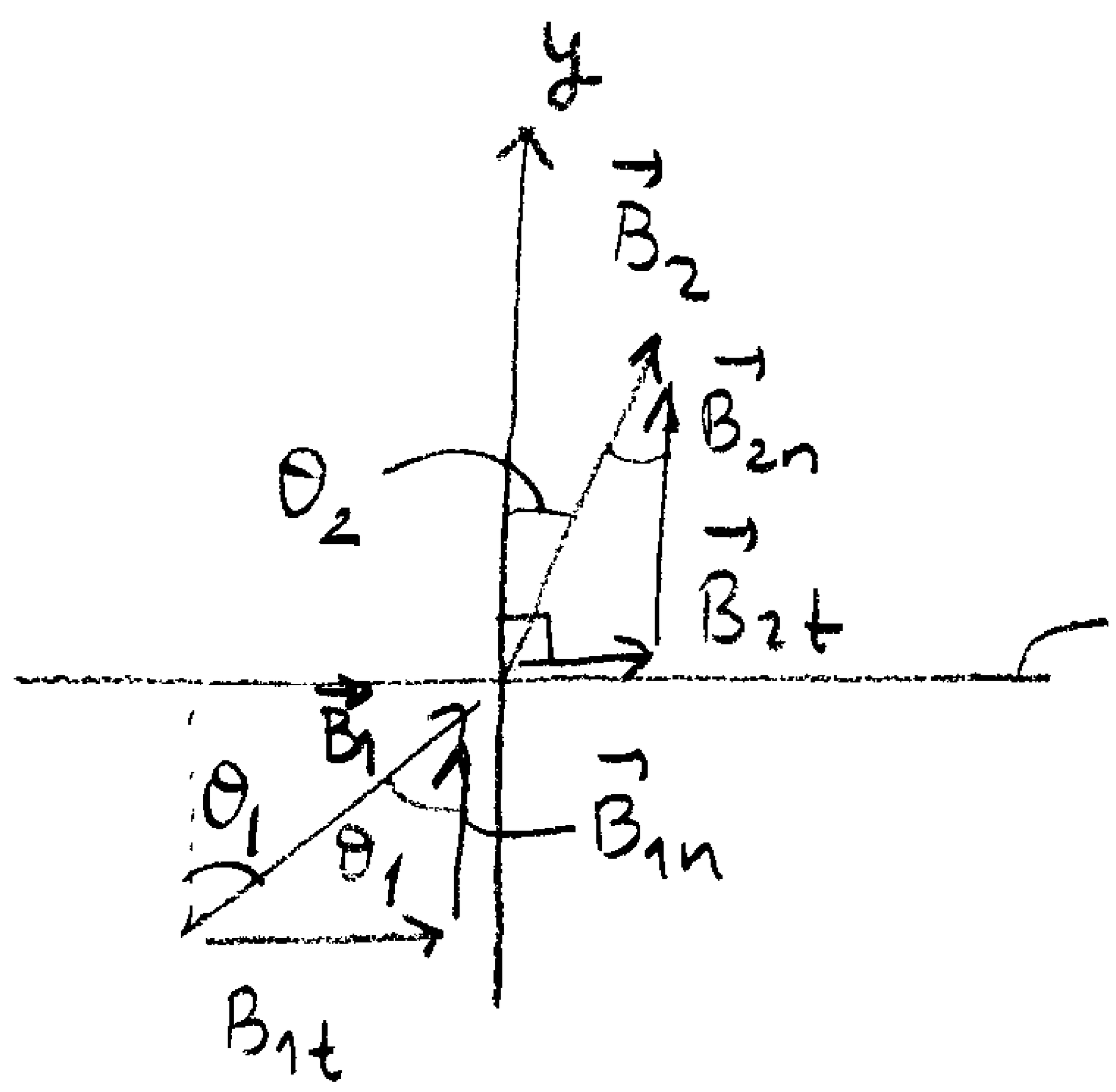
$$\vec{H}_o = 12{,}51\,\vec{a}_x - 12{,}5\,\vec{a}_y \;\; A/m$$

#3)

② $\mu_{r_2} = 1$



① $\mu_{r_1} = 2$

$$\vec{B}_1 = (40\vec{a}_x + 50\vec{a}_y - 30\vec{a}_z) \ \mathrm{mWb/m^2}$$

a) $\vec{B}_{1n} = 50\vec{a}_y$ mT, $\vec{B}_{1t} = 40\vec{a}_x - 30\vec{a}_z$ mT

$\vec{B}_{1n} = \vec{B}_{2n} = 50\vec{a}_y$ mT

Since $\vec{K} = 0$ on the boundary surface

$$\vec{H}_{1t} = \frac{1}{2\mu_0}(40\vec{a}_x - 30\vec{a}_z) = \vec{H}_{2t}$$

$$\vec{B}_{2t} = \mu_0 \vec{H}_{2t} = 20\vec{a}_x - 15\vec{a}_z \ \mathrm{mT}$$

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$$\vec{B}_2 = \vec{B}_{2n} + \vec{B}_{2t} = 20\vec{a}_x + 50\vec{a}_y - 15\vec{a}_z \ \mathrm{mT}$$

b) $\vec{H}_1 = \frac{1}{2\mu_0}(40\vec{a}_x + 50\vec{a}_y - 30\vec{a}_z)^{\mathrm{mA/m}} \rightarrow H_1 = \frac{1}{2\mu_0}[40^2 + 50^2 + 30^2]^{1/2}$

$\vec{H}_2 = \frac{1}{\mu_0}(20\vec{a}_x + 50\vec{a}_y - 15\vec{a}_z)_{\mathrm{mA/m}} \rightarrow H_2 = \frac{1}{\mu_0}[20^2 + 50^2 + 15^2]^{1/2}$

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$$\frac{H_1}{H_2} = \frac{\sqrt{40^2 + 50^2 + 30^2}}{2\sqrt{20^2 + 50^2 + 15^2}} = \frac{\sqrt{5000}}{2\sqrt{3125}} = \frac{1}{2}\sqrt{\frac{5000}{3125}} = 0{,}63245$$

c) $$\tan\theta_1 = \frac{B_{1t}}{B_{1n}} = \frac{[40^2 + 30^2]^{1/2}}{50} = \frac{50}{50} = 1$$

$$\tan\theta_2 = \frac{\vec{B}_{2t}}{\vec{B}_{2n}} = \frac{[20^2 + 15^2]^{1/2}}{50} = \frac{25}{50} = \frac{1}{2}$$

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$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r_1}}{\mu_{r_2}} = \frac{2}{1} = \frac{1}{1/2} = 2$$ Correct!