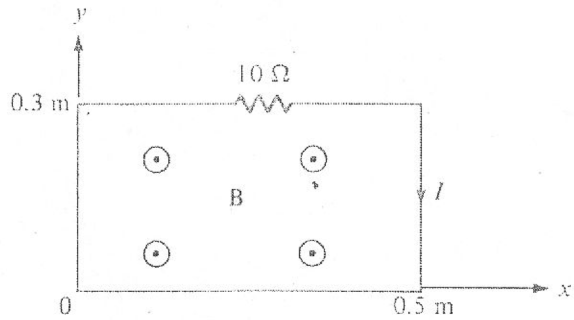
ELECTROMAGNETICS II SECOND EXAM

Dr. Salih FADIL **August 13, 2010**

#1) Find the current I in the following circuit if the circuit is located in a time varying magnetic field

a) $\vec{B} = 0.05t\,\vec{a}_z \;\; Wb/m^2$

b) $\vec{B} = 5\sin(120\pi t - 2y)\,\vec{a}_z \;\; mWb/m^2$



#2) Let $\mu_1 = \mu_0$ for region 1 ($\rho \geq 10$), and $\mu_2 = 3\mu_0$ for region 2 ($\rho \leq 10$). If $\vec{H}_2 = 4\vec{a}_\rho - 6\vec{a}_\phi + 7\vec{a}_z \;\; kA/m$, find:

a) $\vec{H}_1$

b) the angles $\vec{H}_1$ and $\vec{H}_2$ make with the *tangent* to the surface $\rho = 10\ m$.

#3) A 1 C point charge experiences a force of $10\vec{a}_x - 5\vec{a}_y - 7\vec{a}_z\ N$ in a uniform magnetic field when it moves with a velocity of $\vec{a}_x + 2\vec{a}_y\ m/s$ while it experiences a force of $-4\vec{a}_x - 6\vec{a}_y - 2\vec{a}_z\ N$ at a velocity of $\vec{a}_y - 3\vec{a}_z\ m/s$. Calculate the magnetic flux density of the field.

GOOD LUCK.....☺

ELECTROMAGNETICS – II SECOND EXAM

SOLUTION MANUAL

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#1) $V_{emf} = \dfrac{d\lambda}{dt} = \dfrac{d\Psi}{dt} \qquad \Psi = \int_S \vec{B} \cdot d\vec{S}$

a) $\vec{B} = 0.05t\, \vec{a}_z$ T, $\quad d\vec{S} = dx\, dy\, \vec{a}_z$

$$\Psi = 0.05t \int_{y=0}^{0.5}\int_{x=0}^{0.3} dx\, dy = 0.05t\,(0.3 \times 0.5) = 7.5\, 10^{-3}\, t \text{ Wb.}$$

$$V_{emf} = \frac{d\Psi}{dt} = 7.5\, 10^{-3} \text{ Wb}$$

$$I = \frac{7.5\, 10^{-3}}{10} = 0.75 \text{ mA}$$ in the direction shown in the figure

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b) $\Psi = \int_S 5 \sin(120\pi t - 2y)\, \vec{a}_z \cdot dx\, dy\, \vec{a}_z$

$$= \int_{x=0}^{0.5} dx\ 5\int_{y=0}^{0.3} \sin(120\pi t - 2y)\, dy \cdot (+2)\frac{1}{(+2)}$$

$$= x\Big|_0^{0.5} \frac{5}{2}\cos(120\pi t - 2y)\Big|_{y=0}^{0.3}$$

$$= 0.5\ \frac{5}{2}\left[\cos(120\pi t - 0.6) - \cos(120\pi t)\right]$$

$$V_{emf} = \frac{d\Psi}{dt} = 1.25\left\{-\sin(120\pi t - 0.6)\,(120\pi) + 120\pi \sin(120\pi t)\right\}$$ mV

$$I = \frac{1.25 \cdot 120\pi}{10}\left\{-\sin(120\pi t - 0{,}6) + \sin(120\pi t)\right\}$$

$$I = 47.1239\left\{-\sin(120\pi t - 0{,}6) + \sin(120\pi t)\right\} \text{ mA.}$$

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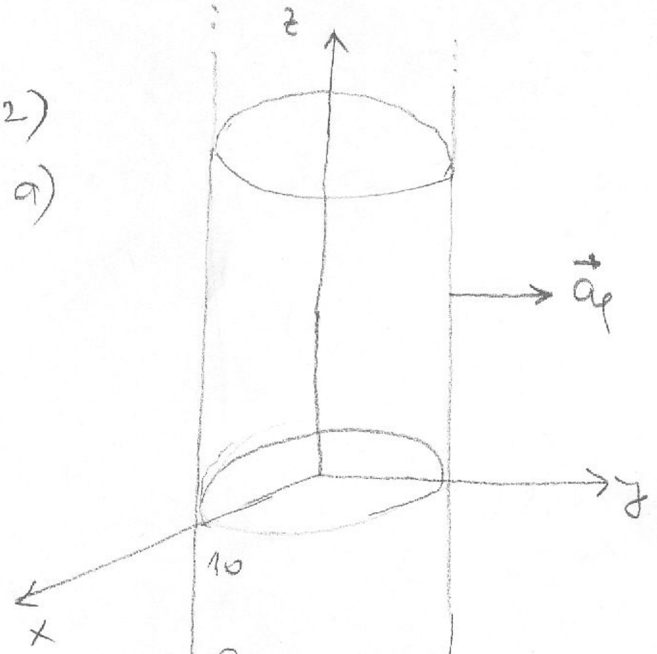
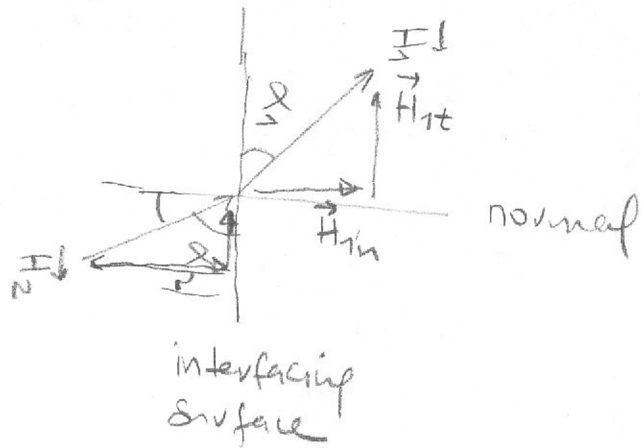
#2)

a)





Region-2

$\mu_2 = 3\mu_0$

Region-1

$\mu_1 = \mu_0$

$$\vec{H}_2 = 4\vec{a}_\rho - 6\vec{a}_\phi + 7\vec{a}_z$$

$$\vec{H}_2 = 4\vec{a}_\rho - 6\vec{a}_\phi + 7\vec{a}_z, \qquad \vec{H}_{2n} = 4\vec{a}_\rho, \quad \vec{H}_{2t} = -6\vec{a}_\phi + 7\vec{a}_z$$

Since $\vec{K} = 0$ on the boundary surface

$$\mu_{r1}\vec{H}_{1n} = \mu_{r2}\vec{H}_{2n} \qquad \vec{H}_{1n} = \frac{3}{1}(4\vec{a}_\rho) = 12\vec{a}_\rho$$

$$\vec{H}_{1t} = \vec{H}_{2t} = -6\vec{a}_\phi + 7\vec{a}_z$$

$$\boxed{\vec{H}_1 = 12\vec{a}_\rho - 6\vec{a}_\phi + 7\vec{a}_z \text{ kA/m}}$$

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b) $\tan\alpha_1 = \dfrac{H_{1n}}{H_{1t}} = \dfrac{12}{(36+49)^{1/2}} \longrightarrow \alpha_1 = 52.46^\circ$

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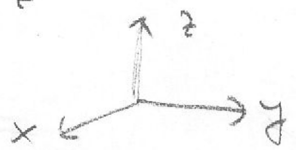
$\tan\alpha_2 = \dfrac{H_{2n}}{H_{2t}} = \dfrac{4}{(36+49)^{1/2}} \longrightarrow \alpha_2 = 23{,}45^\circ$

#3) $\vec{F}_m = Q(\vec{u} \times B)$ $\qquad \vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$

$$10\vec{a}_x - 5\vec{a}_y - 7\vec{a}_z = 1\,(\vec{a}_x + 2a_y) \times (B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z)$$
$$= B_y \vec{a}_z - B_z \vec{a}_y - 2B_x \vec{a}_z + 2B_z \vec{a}_x$$



$$10\vec{a}_x - 5\vec{a}_y - 7\vec{a}_z = 2B_z \vec{a}_x - B_z \vec{a}_y + (B_y - 2B_x)\vec{a}_z$$

$$\left.\begin{array}{l} 10 = 2B_z \\ -5 = -B_z \end{array}\right\} \rightarrow B_z = 5$$

$B_y - 2B_x = -7. \rightarrow$ need one more independent equation

$$-4\vec{a}_x - 6\vec{a}_y - 2\vec{a}_z = (\vec{a}_y - 3\vec{a}_z) \times (B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z)$$
$$= B_x(-\vec{a}_z) + B_z \vec{a}_x - 3B_x \vec{a}_y + -3B_y(-\vec{a}_x)$$
$$= (B_z + 3B_y)\vec{a}_x - 3B_x \vec{a}_y - B_x \vec{a}_z$$

$-4 = B_z + 3B_y$

$-6 = -3B_x \rightarrow \boxed{B_x = 2}$

$-2 = -B_x$

$B_y = -7 + 2B_x$

$\boxed{B_y = -7 + 4 = -3}$

$$\boxed{\vec{B} = 2\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z}$$