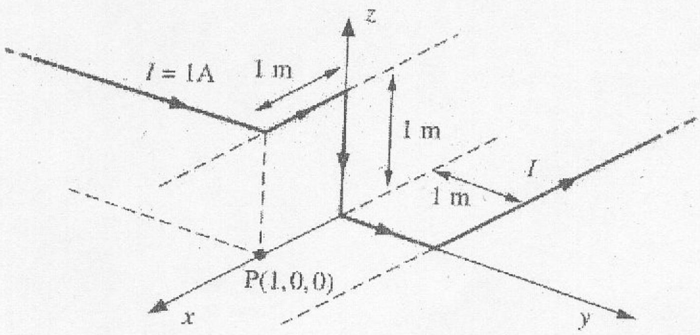
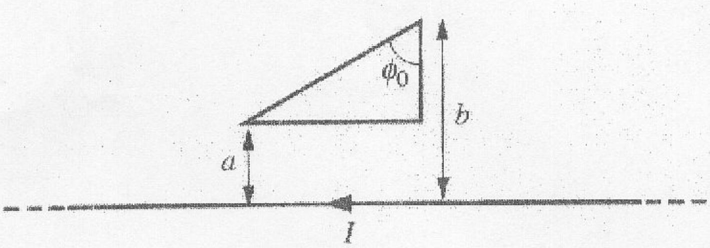
ELECTROMAGNETICS – II SECOND MIDTERM EXAM

Dr. Salih FADIL **December 11, 2009**



#1) An infinitely long wire carrying current $I = 1A$ has four sharp 90^0 bends 1 m apart as shown in the figure. Find the $\vec{B}$ at point $P(1,0,0)$.



#2) A long straight wire carrying a current $I = 100\ A$ and a triangular loop of wire are shown in the figure. Calculate magnetic flux ψ that links the triangular loop for $b = 4a = 40\ cm$ and $\phi_0 = 45^0$

#3) A particle with mass 1 kg and charge 2 C starts from the rest at point $(2, 3, -4)$ in a region where $\vec{E} = -4\vec{a}_y\ V/m$ and $\vec{B} = 5\vec{a}_x\ Wb/m^2$. Calculate:

a) the location of the particle at $t = 1\ s$.

b) Its velocity and K.E at that location.

ELECTROMAGNETICS-II SECOND MIDTERM EXAM SOLUTION MANUAL

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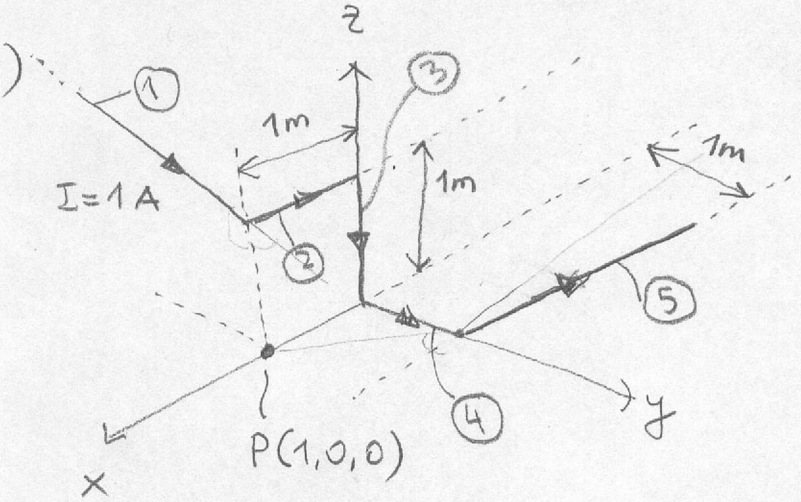
#1)



$\vec{B}(1,0,0) = ?$

$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\vec{a}_\phi \qquad \vec{a}_\phi = \vec{a}_\ell \times \vec{a}_\rho$$

(source → field)

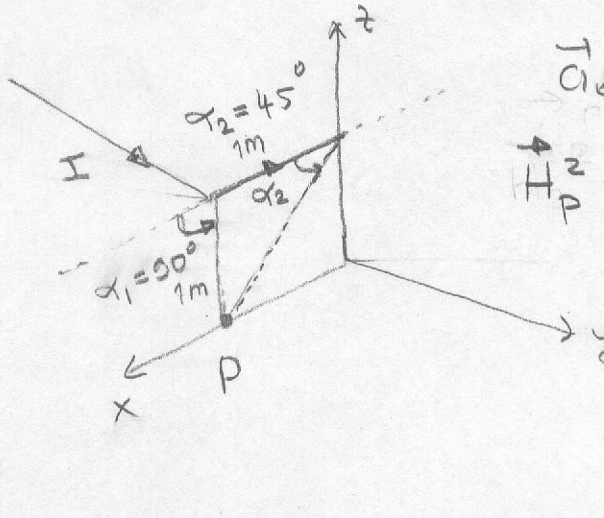
$$\vec{H} = \frac{I}{4\pi\rho}\vec{a}_\phi$$; semi-infinite filamentary current

For portion-1

$$\vec{H}_p^1 = \frac{I}{4\pi\times 1}\vec{a}_\phi \qquad \vec{a}_\phi = \vec{a}_y \times (-\vec{a}_z) = -\vec{a}_x$$

$$\vec{H}_p^1 = \frac{-1}{4\pi}\vec{a}_x \text{ A/m}$$

For portion - 2 $\quad \vec{H}_p^2 = \dfrac{I}{4\pi\rho}(\cos 45^\circ - \cos 90^\circ)\,\vec{a}_\phi$

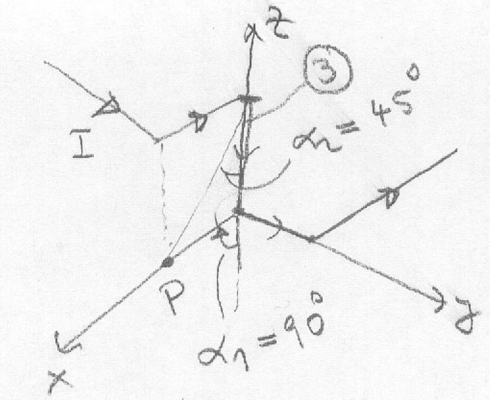


$$\vec{a}_\phi = (-\vec{a}_x)\times(-\vec{a}_z) = -\vec{a}_y$$

$$\vec{H}_p^2 = \frac{1}{4\pi\times 1}\left(\frac{\sqrt{2}}{2} - 0\right)(-\vec{a}_y)$$

$$= \frac{-\sqrt{2}}{8\pi}\vec{a}_y \text{ A/m}$$

For portion - 3

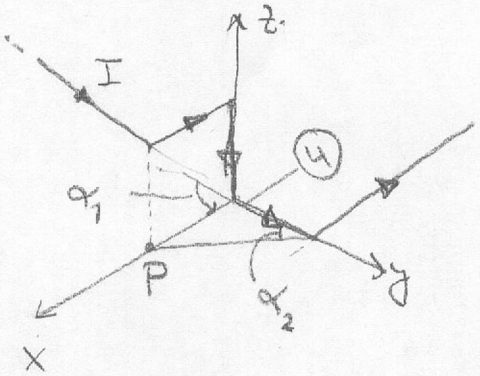


$$\vec{H}_p^3 = \frac{1}{-4\pi}\left(\cos 45° - 0\right)\vec{a}_\phi$$

$$\vec{a}_\phi = (-\vec{a}_z) \times (\vec{a}_x) = -\vec{a}_y$$

$$\vec{H}_p^3 = \frac{1}{4\pi}\left(\frac{\sqrt{2}}{2}\right)(-\vec{a}_y) = \frac{-\sqrt{2}}{8\pi}\vec{a}_y \text{ A/m}$$

For portion - 4

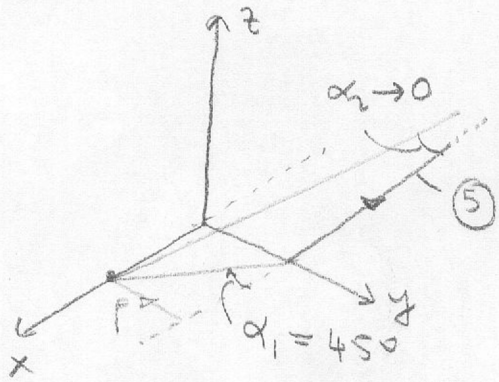


$$\vec{H}_p^4 = \frac{1}{4\pi \times 1}\left(\cos 45° - 0\right)\vec{a}_\phi$$

$$\vec{a}_\phi = (\vec{a}_y) \times (\vec{a}_x) = -\vec{a}_z$$

$$\vec{H}_p^4 = \frac{-\sqrt{2}}{8\pi}\vec{a}_z \text{ A/m}$$

For portion - 5



$$\vec{H}_p^5 = \frac{1}{4\pi \times 1}\left(1 - \frac{\sqrt{2}}{2}\right)\vec{a}_\phi$$

$$\vec{a}_\phi = (-\vec{a}_x) \times (-\vec{a}_y) = \vec{a}_z$$

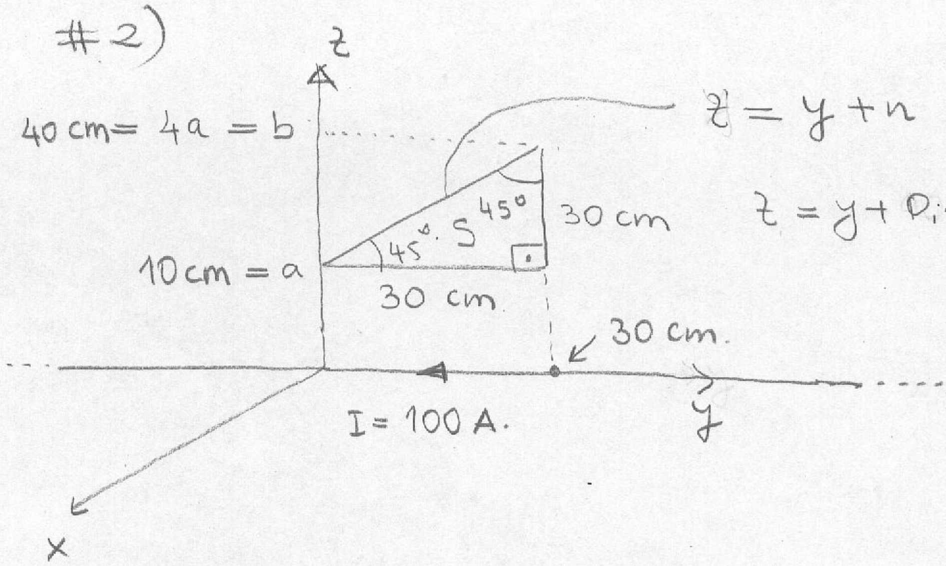
$$\vec{H}_p^5 = \frac{0,2929}{4\pi}\vec{a}_z$$

$$\vec{H}_p = -\frac{1}{4\pi}\vec{a}_x + \frac{\sqrt{2}}{4\pi}\vec{a}_y + \left(\frac{-\sqrt{2}}{8\pi} + \frac{0,2929}{4\pi}\right)\vec{a}_z$$

$$\vec{H}_p = \frac{1}{4\pi}\left(-\vec{a}_x - \sqrt{2}\,\vec{a}_y + 0,4142\,\vec{a}_z\right) \text{ A/m.}$$

$$\vec{B}_p = \underbrace{4\pi\,10^{-7}}_{\mu_0}\vec{H}_p = \left(-\vec{a}_x - \sqrt{2}\,\vec{a}_y - 0,4142\,\vec{a}_z\right)10^{-7} \text{ Wb/m}^2$$

#2)



$z = y + n$ $\qquad a = n$

$z = y + 0{,}1\ m \;,\; y = z - 0{,}1$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi \rho}\vec{a}_\phi \qquad \rho = z \;,\; \vec{a}_\phi = -\vec{a}_y \times (+\vec{a}_z)$$

$$\vec{a}_\phi = -\vec{a}_x$$

$$\vec{B} = \frac{\mu_0 I}{2\pi z}(-\vec{a}_x) \ \text{Wb/m}^2$$

$$\psi = \int_S \vec{B}\cdot d\vec{S} = \int_S \frac{\mu_0 I}{2\pi z}(-a_x)\cdot(dy\ dz)(-\vec{a}_x)$$

$$= \frac{\mu_0 I}{2\pi}\int_{z=0{,}1}^{0{,}4}\frac{1}{z}\left[\int_{y=z-0{,}1}^{0{,}3} dy\right]dz = \frac{\mu_0 I}{2\pi}\int_{z=0{,}1}^{0{,}4}\left[0{,}3-(z-0{,}1)\right]\frac{dz}{z}$$

$$= \frac{\mu_0 I}{2\pi}\int_{z=0{,}1}^{0{,}4}\left(\frac{(0{,}4-z)}{z}\right)dz = \frac{\mu_0 I}{2\pi}\left\{0{,}4\ \ln z\Big|_{0{,}1}^{0{,}4} - z\Big|_{0{,}1}^{0{,}4}\right\}$$

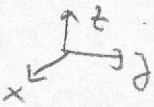
$$= \frac{\cancel{4\pi}\ 10^{-7}\ 100}{\cancel{2\pi}}\left\{0{,}4\ \ln\left(\frac{0{,}4}{0{,}1}\right) - 0{,}3\right\}$$

$$\psi = 5{,}090\ 10^{-6}\ \text{Wb}$$

$$\boxed{\psi = 5{,}090\ \mu\text{Wb}}$$

#3)

a) $m \dfrac{d\vec{u}}{dt} = Q[\vec{E} + \vec{u} \times \vec{B}]$



$$\frac{d}{dt}[u_x \vec{a}_x + u_y \vec{a}_y + \vec{u}_z \vec{a}_z] = \frac{Q}{m}[-4\vec{a}_y + (u_x \vec{a}_x + u_y \vec{a}_y + u_z \vec{a}_z) \times 5\vec{a}_x]$$

$$\frac{d\vec{u}}{dt} = 2[-4\vec{a}_y - 5u_y \vec{a}_z + 5.u_z a_y]$$

$$\frac{du_x}{dt} = 0, \quad \frac{du_y}{dt} = 2[-4 + 5u_z] = -8 + 10u_z, \quad \frac{du_z}{dt} = -10\vec{u}_y$$

$$\frac{d^2u_y}{dt^2} = 10\frac{du_z}{dt} = +10(-10u_y) = -100u_y$$

$$r^2 = -100 \qquad r_{1,2} = \pm j10$$

$$\boxed{u_y(t) = A_1 \cos(10t) + A_2 \sin(10t)}$$

$$u_z(t) = \left(\frac{du_y}{dt} + 8\right)\frac{1}{10} = \frac{1}{10}(8 - 10A_1 \sin(10t) + 10A_2 \cos(10t))$$

$$\boxed{u_z(t) = 0{,}8 - A_1 \sin(10t) + A_2 \cos(10t)}$$

$$\boxed{u_x(t) = A_3}$$

$u_y(0) = 0 = A_1 + 0$, $\boxed{A_1 = 0}$

$u_z(0) = 0 = 0.8 + A_2$ $\boxed{A_2 = -0{,}8}$

$u_x(0) = 0 = A_3$ $\boxed{A_3 = 0}$

$u_x(t) = 0$

$u_y(t) = -0{,}8 \sin(10t)$

$u_z(t) = 0{,}8 - 0{,}8 \cos(10t)$

$x(t) = C_1$

$y(t) = \dfrac{0{,}8}{10}\cos(10t) + C_2$

$z(t) = 0{,}8t - \dfrac{0{,}8}{10}\sin(10t) + C_3$

$x(0) = \boxed{2 = C_1}$

$y(0) = 3 = 0{,}08 + C_2$ $\boxed{C_2 = 2{,}92}$

$z(0) = -4 = 0 - 0 + C_3$ $\boxed{C_3 = -4}$

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$x(1) = 2$

$y(1) = 0{,}08 \cos(10) + 2{,}92 = 2{,}852$

$z(1) = 0{,}8 - 0{,}08 \sin(10) - 4 = -3{,}1564$

$x(1) = 2$ m

$y(1) = 2{,}852$ m

$z(1) = -3{,}1564$ m

b) $u_x(1) = 0$ m/s

$u_y(1) = -0{,}8 \sin(10) = 0{,}4352$ m/s

$u_z(1) = 0{,}8 - 0{,}8 \cos(10) = 1{,}47125$ m/s

$K.E(1) = \frac{1}{2} 1 \times [(0{,}4352)^2 + (1{,}47125)^2] = 1{,}1769$ J