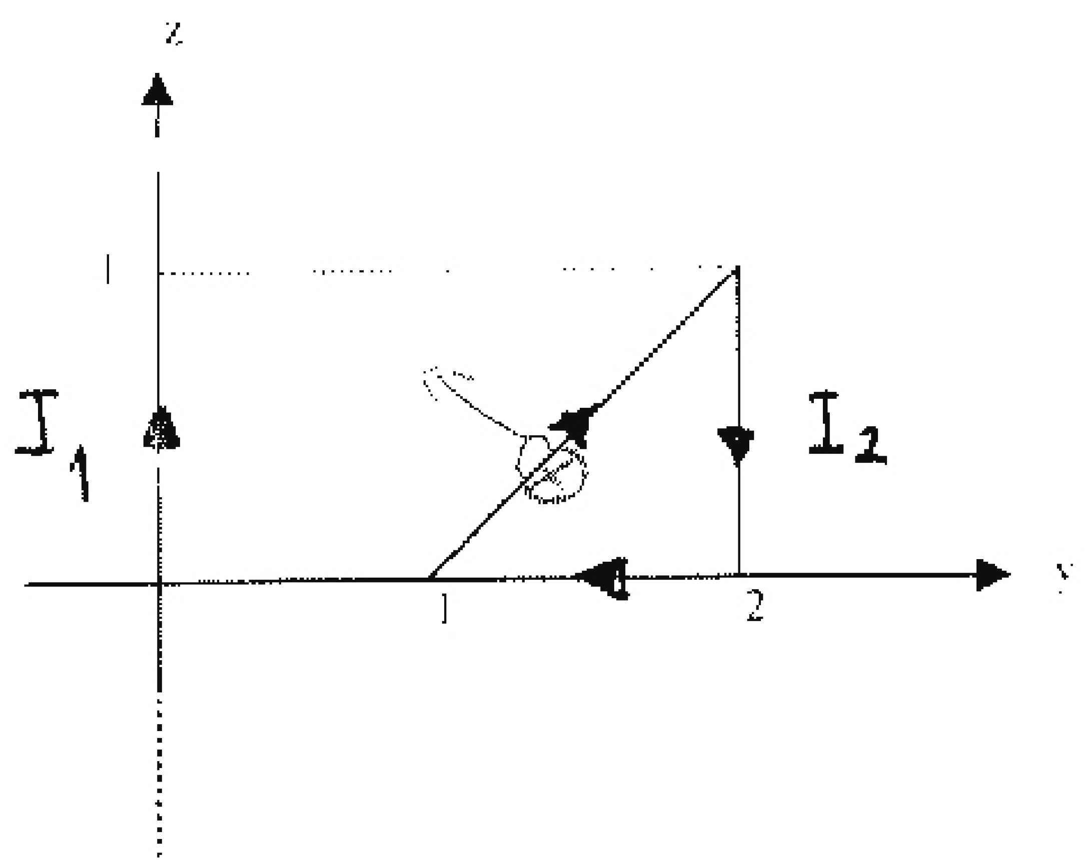
ELECTROMAGNETICS – II SECOND MIDTERM EXAM

- *Two pages A4 sized formula sheet can be used*
- *No problem solution can be written on the formula sheet*
- *Time for the exam is 90 minutes*

Dr. Salih FADIL **December 13, 2010**

#1) In the figure, $I_1 = 10\,A$, $I_2 = 5\,A$, calculate the total force applied on the triangular current loop.



Infinetely long
line current

#2) If $\vec{B}_1 = 2\vec{a}_x - 5\vec{a}_y + 4\vec{a}_z\ mWb/m^2$ in region-1 that is defined by $y + 2x - 2 \le 0$ where $\mu_1 = 3\mu_0$, calculate $\vec{H}_2$ in region-2 that is defined by $y + 2x - 2 \ge 0$ where $\mu_2 = 5\mu_0$.

#3) Given the vector magnetic potential $\vec{A} = e^{-z}\cos y\,\vec{a}_x + (1 + \sin x)\,\vec{a}_z\ Wb/m$ find the magnetic flux through a rectangle loop described by $1 \le x \le 3$, $2 \le y \le 5$, $z = 0$ **<u>only by using</u>** $\vec{B}$ value.

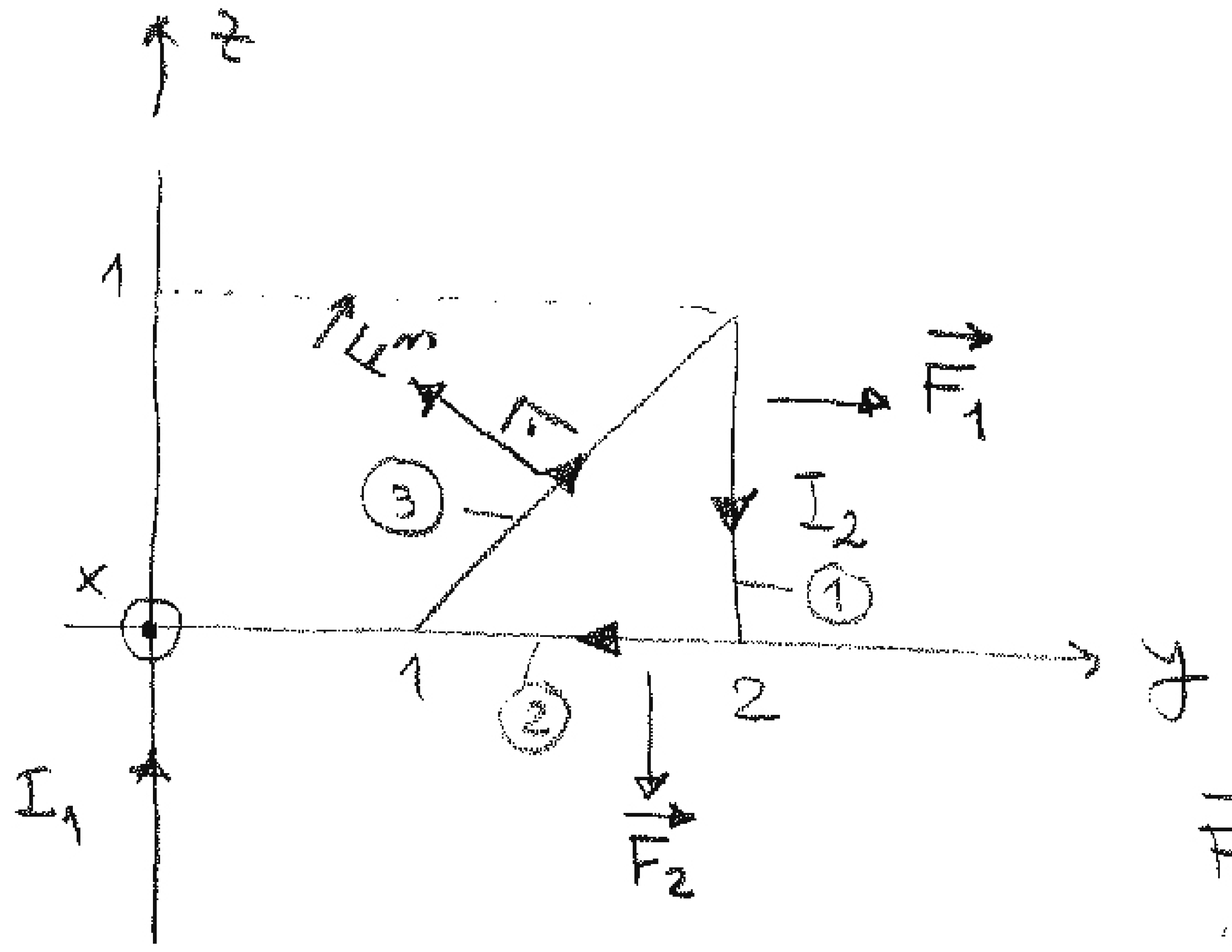
GOOD LUCK....☺

ELECTROMAGNETICS-II SECOND MIDTERM EXAM SOLUTION MANUAL

Dr. Salih FADIL

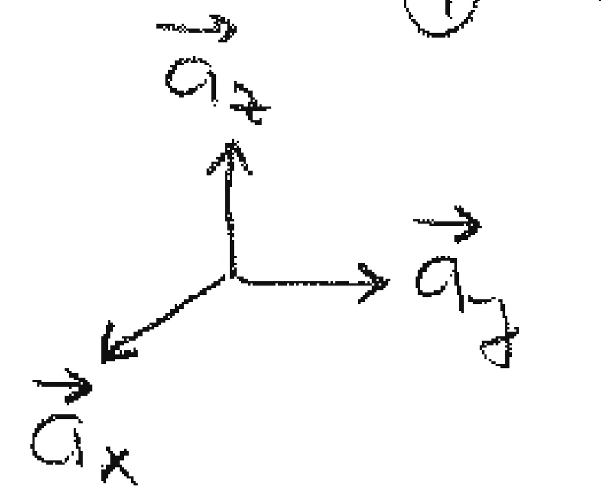
December 13, 2010

#1)



$$\vec{F} = \left(\int_{①} + \int_{②} + \int_{③}\right) I\, \vec{dl} \times \vec{B}$$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho}\vec{a}_\phi = \frac{\mu_0 I_1}{2\pi y}(-\vec{a}_x)$$



For segment - ①

$\vec{dl} = dz\,\vec{a}_z$, $y=2$

$$\vec{F}_1 = \frac{\mu_0 I_1 I_2}{2\pi\, 2}\int_{z=1}^{0} dz\,\vec{a}_z \times (-\vec{a}_x) = \frac{\mu_0 I_1 I_2}{4\pi}\vec{a}_y \left(-z\Big|_1^0\right)$$

$$\vec{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi}\vec{a}_y$$

For segment - ②

$\vec{dl} = dy\,\vec{a}_y$

$$\vec{F}_2 = \frac{\mu_0 I_1 I_2}{2\pi}\int_{y=2}^{1} dy\,\vec{a}_y \times (-\vec{a}_x)\cdot\frac{1}{y} = \frac{\mu_0 I_1 I_2}{2\pi}\vec{a}_z \ln y\Big|_2^1$$

$$\vec{F}_2 = \frac{\mu_0 I_1 I_2}{2\pi}\vec{a}_z(\cancelto{0}{\ln 1} - \ln 2) = -\frac{\mu_0 I_1 I_2}{2\pi}\ln(2)\,\vec{a}_z$$

For segment - ③

$\vec{dl} = dy\,\vec{a}_y + dz\,\vec{a}_z$ $\qquad z = y + n \qquad 0 = n+1 \quad \boxed{n=-1} \qquad z = y-1$

$dz = dy$

$$\vec{F}_3 = \frac{\mu_0 I_1 I_2}{2\pi}\int_{③}\frac{1}{y}\left(dy\,\vec{a}_y + dz\,\vec{a}_z\right)\times(-\vec{a}_x)$$

$$\vec{F}_3 = \frac{\mu_0 I_1 I_2}{2\pi} \int_{3} \frac{1}{z}\Big(dy\,\vec{a}_z + \underbrace{dz}_{= dy}(-\vec{a}_y)\Big) = \frac{\mu_0 I_1 I_2}{2\pi}\left\{\vec{a}_z \int_{y=1}^{2} \frac{dy}{y} - \vec{a}_y \int_{y=1}^{2} \frac{dy}{y}\right\}$$

$$\vec{F}_3 = \frac{\mu_0 I_1 I_2}{2\pi}\left\{(\ln 2 - \cancelto{0}{\ln 1})\,\vec{a}_z + (\cancelto{0}{\ln 1} - \ln 2)\,\vec{a}_y\right\}$$

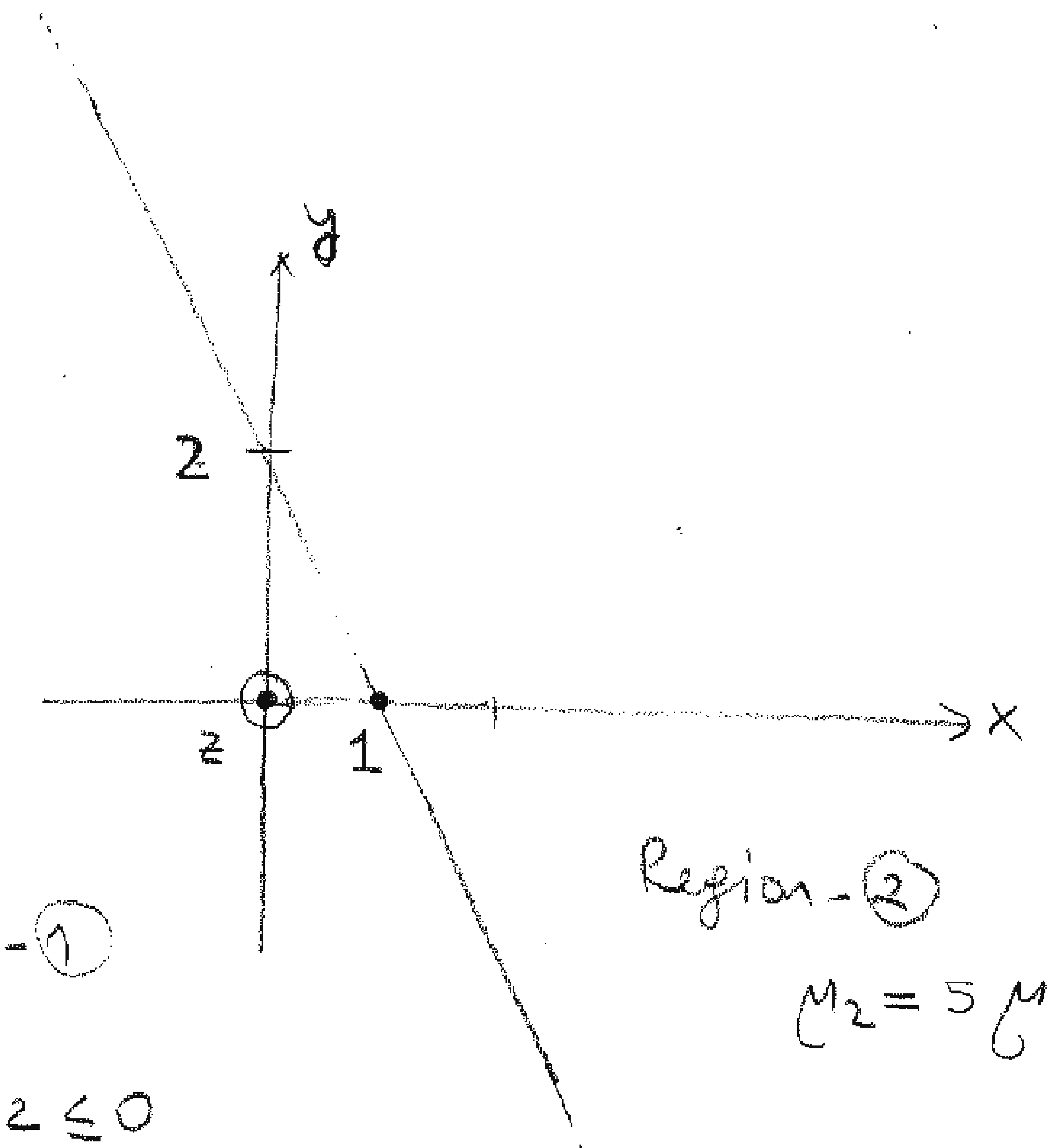
$$\vec{F}_3 = \frac{\mu_0 I_1 I_2}{2\pi} \ln 2\,(\vec{a}_z - \vec{a}_y)$$

$$\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi}\left\{\frac{1}{2}\vec{a}_y - \cancel{\ln 2\,\vec{a}_z} + \cancel{\ln 2\,\vec{a}_z} - \ln 2\,\vec{a}_y\right\}$$

$$\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{1}{2} - \ln 2\right)\vec{a}_y = \frac{\cancelto{2}{4\pi}\,10^{-7}\,10 \times 5}{\cancel{2\pi}}(0{,}5 - \ln 2)\,\vec{a}_y$$

$$\boxed{\vec{F} = -1{,}93147\ 10^{-6}\,\vec{a}_y \ \ \text{N.}}$$

#2)



Region-1

$y + 2x - 2 \leq 0$

$\mu_1 = 3\mu_0$

Region-2

$\mu_2 = 5\mu_0$

$\hookrightarrow y + 2x - 2 = 0,\ z = 0$

interfacing surface

$$f(x,y) = y + 2x - 2$$

$$\nabla f = 2\vec{a}_x + \vec{a}_y$$

$$\vec{a}_n = \pm \frac{\nabla f}{|\nabla f|} = \pm \frac{2\vec{a}_x + \vec{a}_y}{\sqrt{4+1}}$$

$$\vec{a}_n = \frac{1}{\sqrt{5}}(2\vec{a}_x + \vec{a}_y)$$

$$\vec{B}_{1n} = (\vec{B}_{1n} \cdot \vec{a}_n)\,\vec{a}_n$$

$$\vec{B}_{1n} = \left[(2\vec{a}_x - 5\vec{a}_y + 4\vec{a}_z) \cdot \frac{1}{\sqrt{5}}(2\vec{a}_x + \vec{a}_y)\right]\frac{1}{\sqrt{5}}(2\vec{a}_x + \vec{a}_y) \quad \text{mWb/m}^2$$

$$\vec{B}_{1n} = \frac{1}{5}(4-5)\cdot(2\vec{a}_x + \vec{a}_y) = -\frac{1}{5}(2\vec{a}_x + \vec{a}_y) \ \text{mWb/m}^2$$

$$\vec{B}_{1t} = \vec{B}_1 - \vec{B}_{1n} = (2\vec{a}_x - 5\vec{a}_y + 4\vec{a}_z) + \frac{1}{5}(2\vec{a}_x + \vec{a}_y)$$

$$= \left(2 + \frac{2}{5}\right)\vec{a}_x + \left(-5 + \frac{1}{5}\right)\vec{a}_y + 4\vec{a}_z$$

$$\vec{B}_{1t} = \frac{12}{5}\vec{a}_x - \frac{24}{5}\vec{a}_y + 4\vec{a}_z \quad mWb/m^2$$

$$\vec{H}_{2n} = \frac{\mu_1}{\mu_2}\vec{H}_{1n} = \frac{\cancel{\mu_1}}{\mu_2}\frac{\vec{B}_{1n}}{\cancel{\mu_1}} = \frac{1}{5\mu_0}\left[-\frac{1}{5}(2\vec{a}_x + \vec{a}_y)\right]$$

$$\vec{H}_{2n} = -\frac{1}{25\mu_0}(2\vec{a}_x + \vec{a}_y) \quad mA/m$$

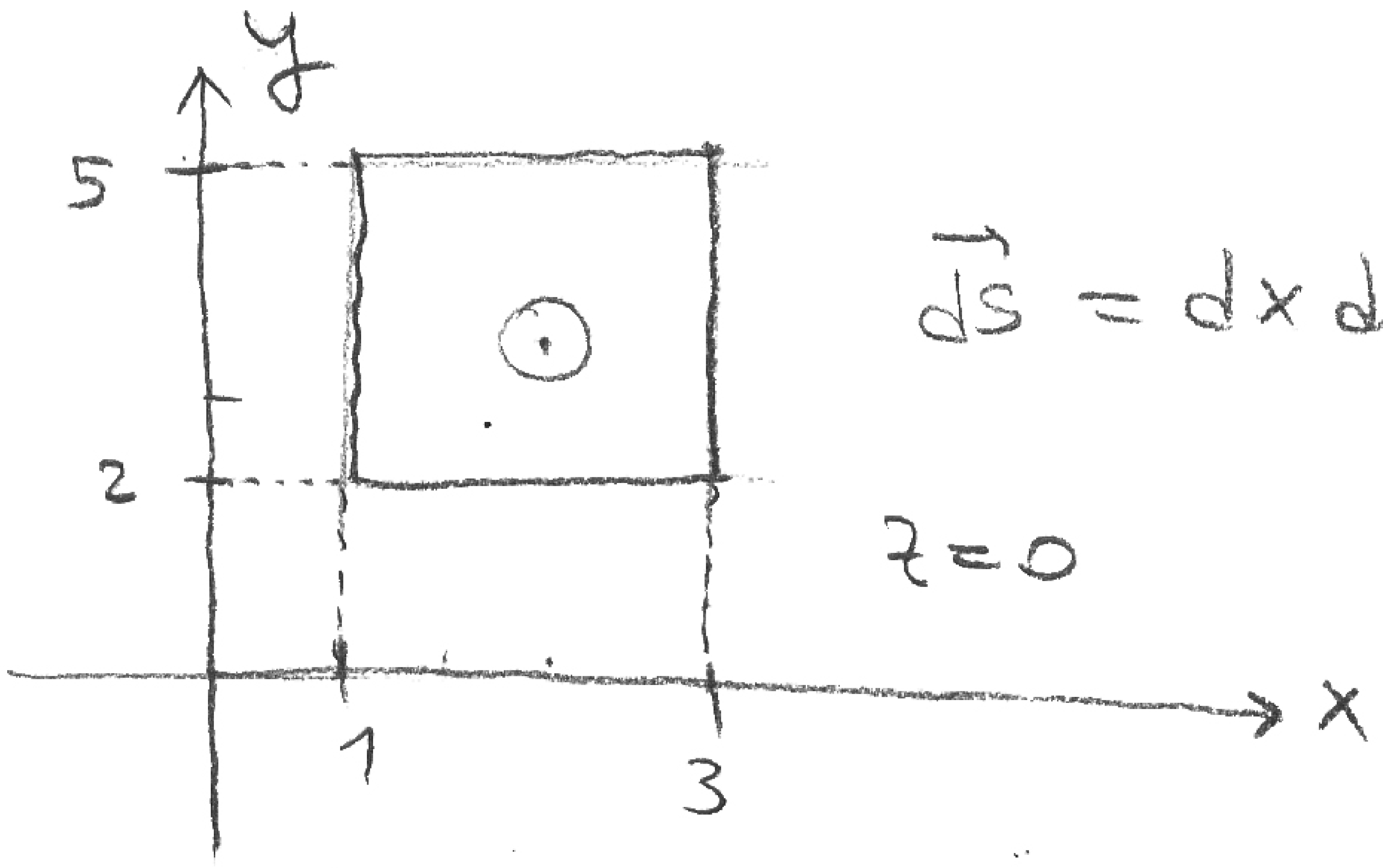
$$\vec{H}_{2t} = \vec{H}_{1t} = \left(\frac{12}{5}\vec{a}_x - \frac{24}{5}\vec{a}_y + 4\vec{a}_z\right)\frac{1}{\mu_1} = \frac{1}{3\mu_0}\left(\frac{12}{5}\vec{a}_x - \frac{24}{5}\vec{a}_y + 4\vec{a}_z\right)$$

$$\vec{H}_2 = \frac{10^{-3}}{\mu_0}\left[\vec{a}_x \underbrace{\left(-\frac{2}{25} + \frac{12}{15}\right)}_{0.72} + \underbrace{\left(-\frac{1}{25} - \frac{24}{15}\right)}_{-1,64}\vec{a}_y + \underbrace{\frac{4}{3}}_{1.33}\vec{a}_z\right] \quad mA/m$$

$$\boxed{\vec{H}_2 = 572.95\,\vec{a}_x - 1305\,\vec{a}_y + 1061\,\vec{a}_z \quad A/m}$$

#3) $\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{-z}\cos y & 0 & (1+\sin x) \end{vmatrix} = \vec{a}_x[(0-0)] - \vec{a}_y[\cos x + e^{-z}\cos y] + \vec{a}_z[0 + e^{-z}\sin y]$

$$\vec{B} = -(\cos x + e^{-z}\cos y]\,\vec{a}_y + e^{-z}\sin y\,\vec{a}_z$$



$\vec{ds} = dx\,dy\,\vec{a}_z$

$z = 0$

$\Psi = x\Big|_1^3 \left(-\cos y\Big|_2^5\right)$

$\Psi = (3-1)(-\cos 5 + \cos 2)$

$\underline{\Psi = -1.4 \text{ Wb}}$ (+ 1.4 Wb in $(-\vec{a}_z)$ direction)

$$\Psi = \int_S \vec{B}\cdot\vec{ds} = \int_S e^0 \sin y\,dx\,dy = \int_1^3 dx \int_{y=2}^5 \sin y\,dy$$