1/ $y' + y = \sin x$, $y(0) = 1$ başlangıç değer problemini Laplace dönüşümü ile çözünüz.

Çözüm: $\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin x\}$, $\mathcal{L}\{y\} = Y(s)$

$$\left(sY(s) - \underbrace{y(0)}_{1}\right) + Y(s) = \frac{1}{s^2+1}$$

$$sY(s) - 1 + Y(s) = \frac{1}{s^2+1}$$

$$(s+1)Y(s) = \frac{1}{s^2+1} + 1$$

$$Y(s) = \frac{1}{(s+1)(s^2+1)} + \frac{1}{s+1}$$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\frac{1}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1}$$

$$1 = A(s^2+1) + (Bs+C)(s+1)$$

$s=-1$ $\quad A = 1/2$

$s=0$ $\quad 1 = A + C \Rightarrow C = 1/2$

$s=1$ $\quad 1 = 2A + (B+C)2$

$\quad 1 = 1 + 2B + 1 \Rightarrow B = -1/2$

$$y(x) = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$= \frac{1}{2}e^{-x} - \frac{1}{2}\cos x + \frac{1}{2}\sin x + e^{-x} = \frac{3}{2}e^{-x} - \frac{1}{2}\cos x + \frac{1}{2}\sin x$$

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2/ $y'' - y' - 2y = 4t^2$, $y(0) = 1$, $y'(0) = 4$ başlangıç değer problemini Laplace dönüşümü ile çözünüz.

Çözüm: $\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = 4\mathcal{L}\{t^2\}$, $\mathcal{L}\{y\} = Y(s)$

$$\left(s^2Y(s) - sy(0) - y'(0)\right) - \left(sY(s) - y(0)\right) - 2Y(s) = 8/s^3$$

$$s^2Y(s) - s - 4 - sY(s) + 1 - 2Y(s) = \frac{8}{s^3}$$

$$Y(s) = \frac{s+3}{s^2-s-2} + \frac{8}{s^3(s^2-s-2)} = \frac{s^4+3s^3+8}{s^3(s^2-s-2)}$$

$$\frac{s^4+3s^3+8}{s^3(s^2-s-2)} = \frac{2}{s-2} + \frac{2}{s+1} - \frac{4}{s^3} + \frac{2}{s^2} - \frac{3}{s}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$y(t) = 2e^{2t} + 2e^{-t} - 2t^2 + 2t - 3$$

3/ $y^{(4)} + 2y'' + y = 0$, $y(0) = y'(0) = y'''(0) = y''(0) = 1$ başlangıç değer problemini Laplace dönüşümü ile çözünüz.

__Çözüm:__ $\mathcal{L}\{y^{(4)}\} + 2\mathcal{L}\{y''\} + \mathcal{L}\{y\} = 0$

$$s^4Y(s) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0) + 2s^2Y(s) - 2sy(0) - 2y'(0) + Y(s) = 0$$

$$(s^4 + 2s^2 + 1)Y(s) = s \Rightarrow Y(s) = \frac{s}{(s^2+1)^2}$$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$$

$$y(x) = \frac{1}{2}t\sin t$$

4/ $ty'' - (t-1)y' - y = 0$, $y(0)=5$ problemini Laplace dönüşümü ile çözünüz.

Çözüm

$$\mathcal{L}\{ty'' - (t-1)y' - y\} = \mathcal{L}\{0\}$$

$$\mathcal{L}\{ty''\} - \mathcal{L}\{ty'\} + \mathcal{L}\{y'\} - \mathcal{L}\{y\} = 0$$

$$-\frac{d}{ds}\left[s^2Y(s) - sy(0) - y'(0)\right] + \frac{d}{ds}\left[sY(s) - y(0)\right] + sY(s) - y(0) - Y(s) = 0$$

$$-2sY(s) - s^2\frac{dY}{ds} + 5 + Y(s) + s\frac{dY}{ds} + sY(s) - 5 - Y(s) = 0$$

$$\frac{dY}{Y(s)} + \frac{1}{s-1}ds = 0$$

$$\Rightarrow \ln Y(s) + \ln(s-1) = \ln c$$

$$Y(s) = \frac{c}{s-1}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = ce^t$$

$$y(0) = 5 \Rightarrow 5 = c$$

$$\Rightarrow y(t) = 5e^t$$

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5/ $\begin{cases} x'(t) = 2x-3y \\ y'(t) = y-2x \end{cases}$, $x(0)=8$, $y(0)=3$ diferansiyel denklem sisteminin Laplace dönüşümü ile çözümünü bulunuz.

Çözüm:

$$\begin{cases} \mathcal{L}\{x'\} = 2\mathcal{L}\{x\} - 3\mathcal{L}\{y\} \\ \mathcal{L}\{y'\} = \mathcal{L}\{y\} - 2\mathcal{L}\{x\} \end{cases}$$

$$\begin{cases} sX(s) - x(0) = 2X(s) - 3Y(s) \\ sY(s) - y(0) = Y(s) - 2X(s) \end{cases}$$

$$\begin{cases} (s-2)X(s) + 3Y(s) = 8 \\ 2X(s) + (s-1)Y(s) = 3 \end{cases}$$

$$X(s) = \frac{8s-17}{s^2-3s-4} = \frac{5}{s+1} + \frac{3}{s-4}$$

$$Y(s) = \frac{3s-22}{s^2-3s-4} = \frac{5}{s+1} - \frac{2}{s-4}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = 5e^{-t} + 3e^{4t}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 5e^{-t} - 2e^{4t}$$

6/ $x''+y=0$, $\quad x'(0)=-2,\ x(0)=0,\ y(0)=0$ denklem

$x''-y'=-2e^t$

sistemini Laplace dönüşüm yöntemi ile çözünüz.

Çözüm:
$$\begin{cases} \mathcal{L}\{x''+y\}=0 \\ \mathcal{L}\{x''-y'\}=\mathcal{L}\{-2e^t\} \end{cases} \Rightarrow \begin{cases} \mathcal{L}\{x''\}+\mathcal{L}\{y\}=0 \\ \mathcal{L}\{x''\}-\mathcal{L}\{y'\}=-2\mathcal{L}\{e^{-t}\} \end{cases}$$

$\mathcal{L}\{x\}=X(s)$, $\quad \mathcal{L}\{y\}=Y(s)$ olmak üzere

$$\begin{cases} s^2X(s)-sX(0)-X'(0)+Y(s)=0 \\ s^2X(s)-sX(0)-X'(0)-sY(s)+y(0)=-2/(s-1) \end{cases}$$

$x'(0)=-2$
$x(0)=0$
$y(0)=0$
$$\begin{cases} s^2X(s)+Y(s)=-2 \\ s^2X(s)-sY(s)=-2-2/(s-1) \end{cases}$$

Taraf tarafa çıkarıldığında

$$Y(s)=\frac{2}{(s+1)(s-1)}=\frac{1}{s-1}-\frac{1}{s+1}$$

$$X(s)=\frac{2}{s^2(s^2-1)}-\frac{2}{s^2}=\frac{-2}{(s+1)(s-1)}=-\frac{1}{s-1}+\frac{1}{s+1}$$

$$\mathcal{L}^{-1}\{Y(s)\}=e^t-e^{-t}=y(t)$$

$$\mathcal{L}^{-1}\{X(s)\}=-e^t+e^{-t}=x(t)$$