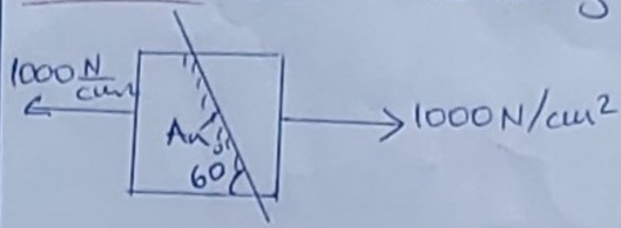
TEK EKSENLİ GERİLME ÖRNEKLERİ

ÖRNEK 1) Şekilde verilen gerilme elemanında; Mohr D. çizerek

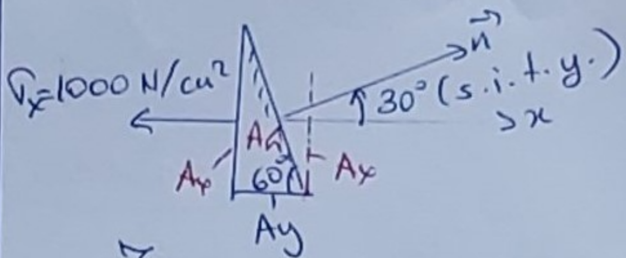


a) $\sigma_n = ?$ $\tau_n = ?$

b) $\sigma_{1,2} = ?$ ve düzlemleri = ?

c) $\tau_{\substack{max\\min}} = ?$ ve düzlemleri

d) Bulduğunuz değerleri formüllerle doğrulayınız.



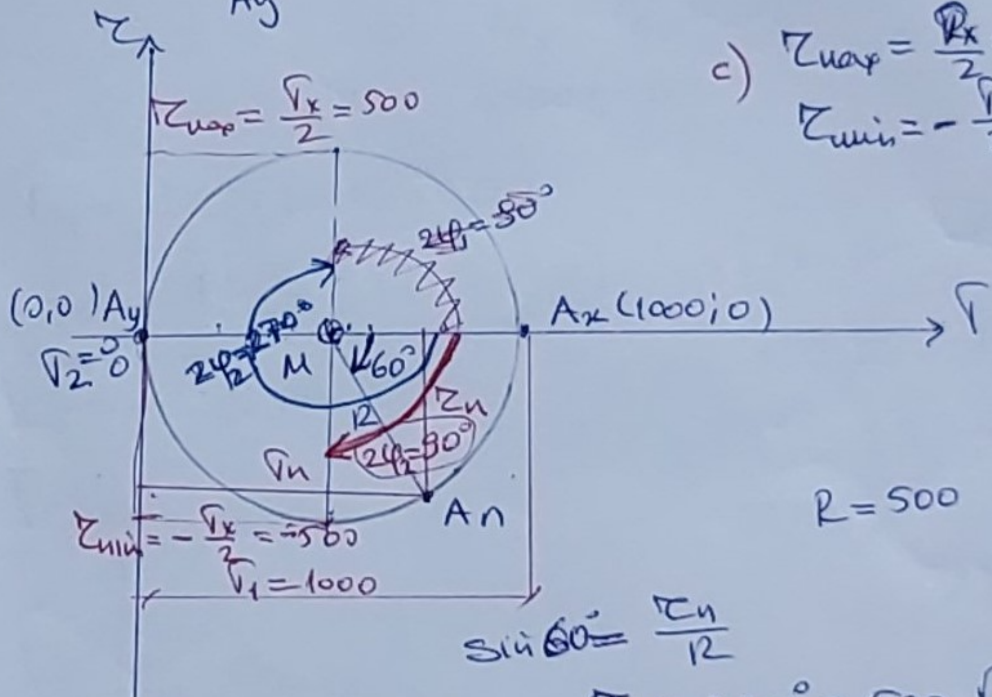
$A_x (1000; 0)$

$A_y (0, 0)$

ölç: 1 cm = 200 N/cm²

c) $\tau_{max} = \frac{\sigma_x}{2} = 500$ N/cm²; $2\varphi_2 = 270°$

$\tau_{min} = -\frac{\sigma_x}{2} = -500$ .. $2\varphi_1 = -90°$



$\sin 60 = \frac{\tau_n}{R}$

R = 500

a) Mohr Dairesinden: $\tau_n = R \sin 60° = 500 \cdot \frac{\sqrt{3}}{2} \Rightarrow \boxed{\tau_n = -433 \frac{N}{cm^2}}$

$\sigma_n = R + R\cos 60° = 500 + 500\left(\frac{1}{2}\right) \Rightarrow \boxed{\sigma_n = 750 \text{ N/cm}^2}$

Hesap yolu ile (formülle)

$\sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta) = \frac{1000}{2}(1+\cos 60°) = 750$ N/cm²

$\tau_n = -\frac{\sigma_x}{2}\sin 2\theta = -\frac{1000}{2}\sin 60°) = -433$ N/cm²

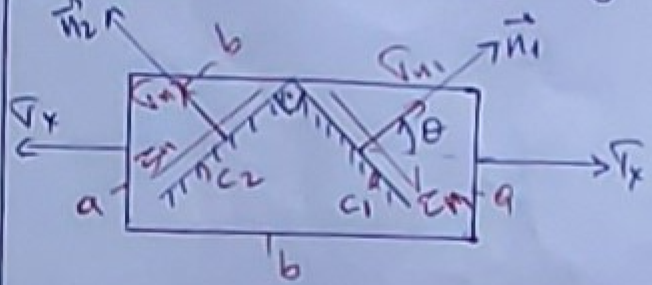
b) Mohr Dairesinden: $\sigma_1 = \sigma_x = 1000$ N/cm² .. (Düzlemi: A_x, $\theta_1 = 0°$) $2\theta_1 = 0°$

$\sigma_2 = 0$ - - - - - - (" A_y, $\theta_2 = 90°$) $2\theta_2 = 180°$

Formüller: $\sigma_1\Big|_{\theta=0°} = \frac{1000}{2}(1+\underset{1}{\cos 0°}) = \boxed{1000 = \sigma_x}$

$\sigma_2\Big|_{\theta=90°} = \frac{1000}{2}(1+\underset{-1}{\cos 180°}) = \boxed{0}$

ÖRNEK 2) Şekildeki gibi yüklü elemanda;



$\sigma_{n_1} = 4500\ N/cm^2$

$\sigma_{n_2} = 1500\ N/cm^2$

a) Mohr Dairesini çiziniz

b) $\sigma_x = ?$ $\theta = ?$ $\tau_A = ?$

c) c_1 ve c_2 nin asal gerilmelerini bulunuz.

d) $\tau_{\substack{max\\min}} = ?$

ÇÖZÜM:

$$c_1 \perp c_2 \Rightarrow \sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2 = \sigma_x$$

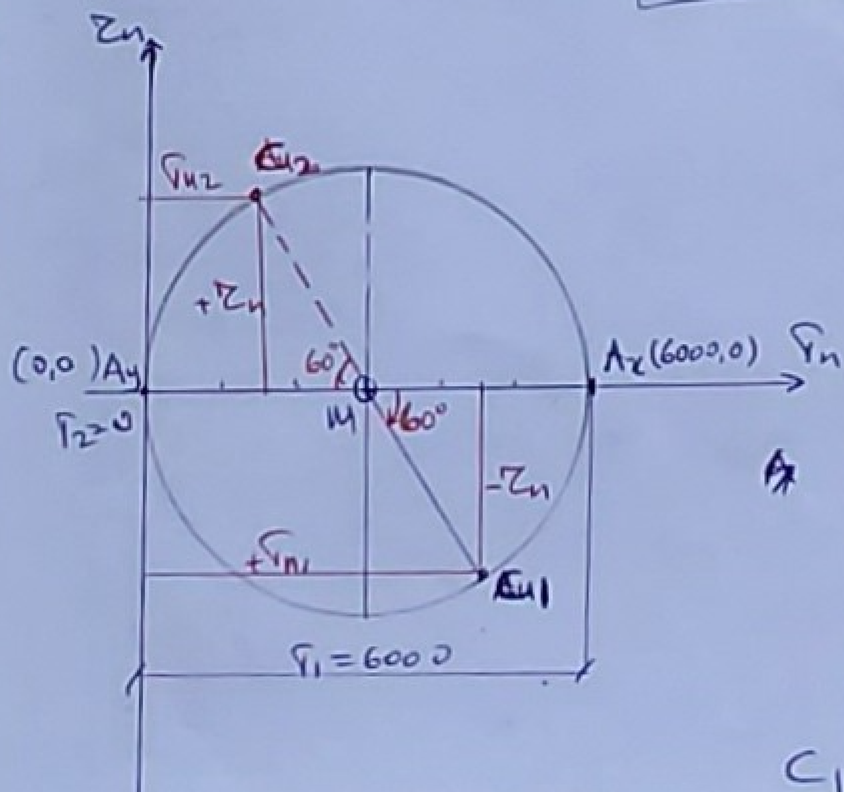
$$4500 + 1500 = \sigma_x \Rightarrow \boxed{\sigma_x = 6000\ N/cm^2}$$

$$\sigma_{n_1} = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

$$4500 = \frac{6000}{2}(1 + \cos 2\theta) \Rightarrow \boxed{\theta = 30^\circ}$$

$$\tau_n = -\frac{\sigma_x}{2}\sin 2\theta \Rightarrow \tau_n = -\frac{6000}{2}\sin 60^\circ$$

$$\boxed{\tau_n = -2598\ N/cm^2}$$



$A_x(\sigma_x ; 0) = (6000 : 0)$

$A_y(0, 0)$

ölç: 1cm = 1000 N/cm²

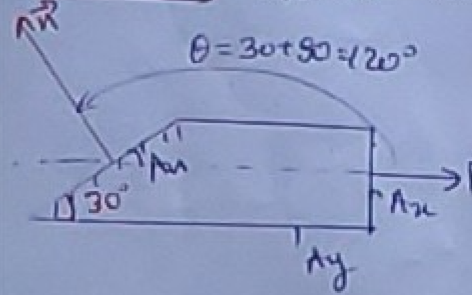
$C_1(\sigma_{n_1} ; -\tau_n)$

$C_1(4500 ; -2598)$

$C_2(1500 ; +2598)$

$C_1 \perp C_2$

ÖRNEK 3) $8{,}0\ cm^2$ kesitli bir çubuğun iki ucuna $P=600N$ çekme kuvveti uygulanmıştır.



a) Çubuğun şekildeki gibi 30° eğik açı yapan eğik düzlemini gerilmelerini bulunuz. ($\sigma_n = ?$, $\tau_n = ?$)

b) Mohr Dairesini çiziniz

ÇÖZÜM: $\theta = 30° + 90° = 120°$ (s.i.t.y.)

a)

$$\sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta) \qquad \sigma_x = \frac{P}{A} = \frac{600}{8{,}0} = 75\ N/cm^2$$

$$\sigma_n = \frac{75}{2}(1+\underbrace{\cos 120°}_{-0{,}5})$$

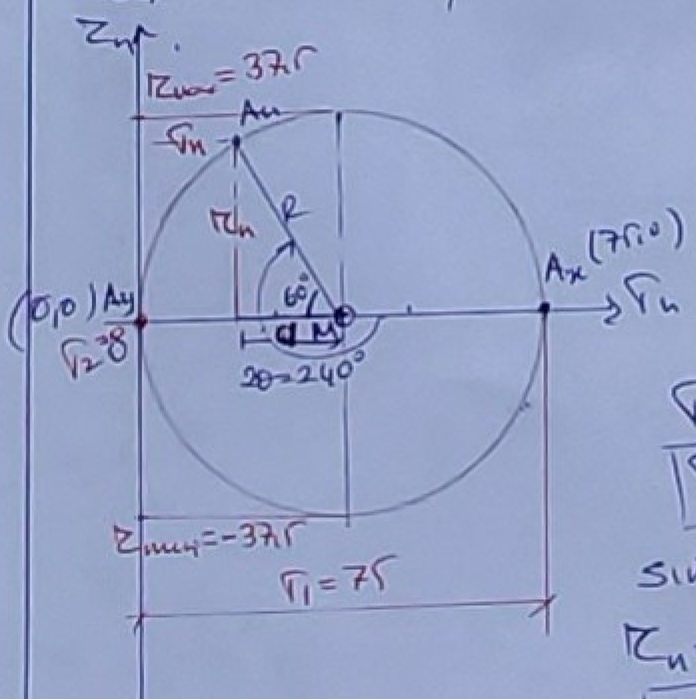
$$\boxed{\sigma_n = 18{,}75\ N/cm^2}$$

$$\tau_n = -\frac{75}{2}\underbrace{\sin 120°}_{0{,}866}$$

$$\boxed{\tau_n = -32{,}5\ N/cm^2}$$

b) MOHR DAİRESİ: $A_x(\sigma_x ; 0) = A_x(75;0)$

ölç: $1cm = 25\ N/cm^2$ $\}$ $A_y(0,0)$



$$R = \frac{\sigma_x}{2} = 37{,}5$$

$$\cos 60° = \frac{a}{R}$$

$$a = R\cos 60°$$

$$a = 37{,}5 \underbrace{\cos 60°}_{1/2}$$

$$a = 18{,}75$$

$$\sigma_n = R - a = 37{,}5 - 18{,}75$$

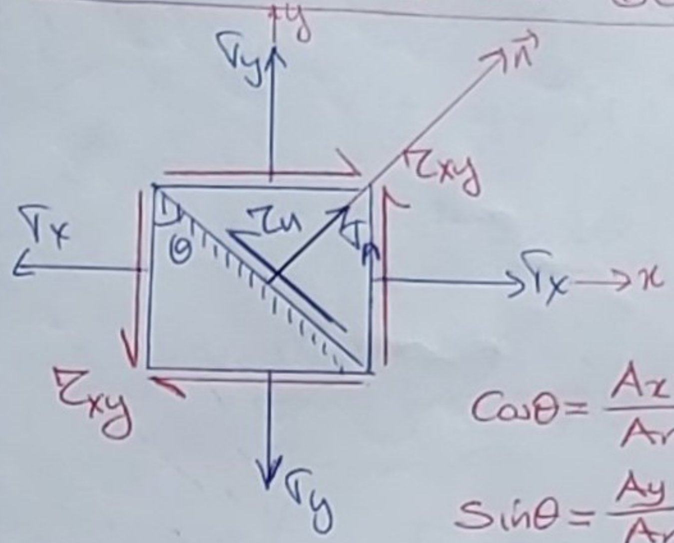
$$\boxed{\sigma_n = 18{,}75\ N/cm^2}$$

$$\sin 60° = \frac{\tau_n}{R}$$

$$\tau_n = 37{,}5 \cdot \sin 60°$$

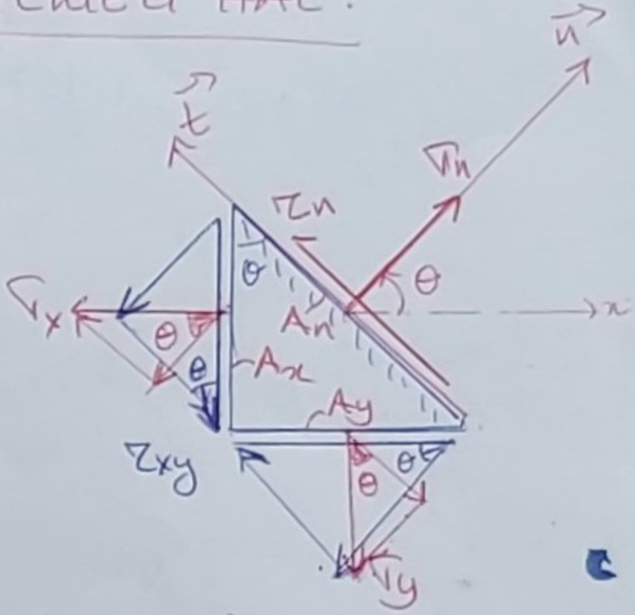
$$\boxed{\tau_n = -32{,}5\ N/cm^2}$$

İKİ EKSENLİ KAYMA GERİLMELİ HAL:



$$\cos\theta = \frac{A_x}{A_n}$$

$$\sin\theta = \frac{A_y}{A_n}$$



$$\Sigma F_n = 0: \ \sigma_n A_n - (\sigma_x \cos\theta) A_x - (\sigma_y \sin\theta) A_y - (\tau_{xy} \sin\theta) A_x - (\tau_{xy} \cos\theta) A_y = 0$$

$$\sigma_n = \sigma_x \overset{\cos\theta}{\left(\frac{A_x}{A_n}\right)} \cos\theta + \sigma_y \overset{\sin\theta}{\left(\frac{A_y}{A_n}\right)} \sin\theta + \tau_{xy} \overset{\cos\theta}{\left(\frac{A_x}{A_n}\right)} \sin\theta + \tau_{xy} \overset{\sin\theta}{\left(\frac{A_y}{A_n}\right)} \cos\theta$$

$$\boxed{\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta} \quad \text{(A)}$$

$$\Sigma F_t = 0: \ \tau_n A_n + (\sigma_x \sin\theta) A_x - (\sigma_y \cos\theta) A_y - (\tau_{xy} \cos\theta) A_x + (\tau_{yx} \sin\theta) A_y = 0$$

$$\tau_n = -\sigma_x \sin\theta \overset{\cos\theta}{\left(\frac{A_x}{A_n}\right)} + \sigma_y \cos\theta \overset{\sin\theta}{\left(\frac{A_y}{A_n}\right)} + \tau_{xy} \overset{\cos\theta}{\left(\frac{A_x}{A_n}\right)} \cos\theta - \tau_{xy} \overset{\sin\theta}{\left(\frac{A_y}{A_n}\right)} \sin\theta$$

$$\boxed{\tau_n = -\sigma_x \cos\theta \sin\theta + \sigma_y \sin\theta \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)} \quad \text{(B)}$$

<u>2θ cinsinden</u>

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta), \qquad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin\theta \cos\theta = \frac{1}{2} \sin 2\theta$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

2θ cinsinden: $\sigma_n = ?$

$$\sigma_n = \sigma_x\left[\frac{1}{2}(1+\cos 2\theta)\right] + \sigma_y\left[\frac{1}{2}(1-\cos 2\theta)\right] + \cancel{2}\tau_{xy}\left[\frac{1}{\cancel{2}}\sin 2\theta\right]$$

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta} \quad \ldots (1)$$

2θ cinsinden: $\tau_n = ?$

$$\tau_n = -\sigma_x\left[\frac{1}{2}\sin 2\theta\right] + \sigma_y\left[\frac{1}{2}\sin 2\theta\right] + \tau_{xy}(\cos 2\theta)$$

$$\boxed{\tau_n = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta} \quad \ldots (2)$$

(1) ve (2) den gerekli kısaltmalar yapılırsa, daha önce çıkartılan tek eksenli gerilmedeki

$$\boxed{\sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta) \quad \text{ve} \quad \tau_n = -\frac{\sigma_x}{2}\sin 2\theta}$$

gerilme değerleri ve de iki eksenlideki kayma gerilmesiz hal formülleri

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta, \quad \tau_n = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta}$$

elde edilir

ASAL GERİLMELER VE DÜZLEMLERİNİN BULUNMASI:

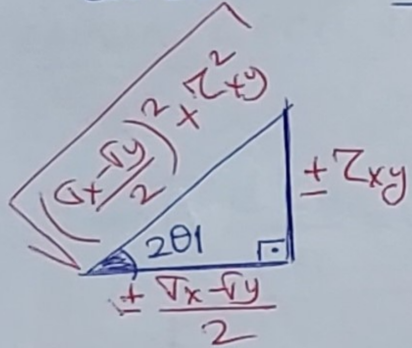
① den, $\dfrac{d\sigma_n}{d\theta} = -2\,\dfrac{(\sigma_x-\sigma_y)}{2}\sin 2\theta + 2\,\tau_{xy}\cos 2\theta = 0$

$\dfrac{d(\sin 2\theta)}{d\theta} = +2\cos 2\theta$

$\dfrac{d(\cos 2\theta)}{d\theta} = -2\sin 2\theta$

$$(\sigma_x-\sigma_y)\sin 2\theta = 2\,\tau_{xy}\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}} = \tan 2\theta_1 \Rightarrow \boxed{\tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}} \quad (3)$$



Asal gerilme değerlerini veren bu açı değerleri ① de konursa

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\underbrace{\frac{\left(\pm\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}}_{\cos 2\theta} + \tau_{xy}\underbrace{\frac{(\pm\tau_{xy})}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}}_{\sin 2\theta}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \mp \frac{1}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}\left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2\right]$$

$$\boxed{\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}} \quad \text{---}(4)$$

Asal gerilme düzlemlerinde kayma gerilmesinin olmadığını göstermek için, ③ formülüyle verilen ve aralarında 90° bulunan açılar ② de yerine konur,

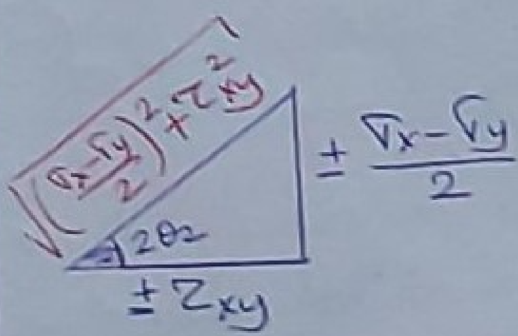
$$\tau_n = -\frac{\sigma_x-\sigma_y}{2}\underbrace{\left[\frac{\pm\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}\right]}_{\sin 2\theta} + \tau_{xy}\underbrace{\left[\frac{\left(\pm\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}\right]}_{\cos 2\theta}$$

$$\boxed{\tau_n = 0}$$

KAYMA GERİLMELERİNİN MAKSİMUM VE MİNİMUM DEĞERLERİNİN VE DÜZLEMLERİNİN BULUNMASI:

(2) den: $\frac{d\tau_n}{d\theta} = 0 \Rightarrow -\cancel{2}\,\frac{\sigma_x-\sigma_y}{\cancel{2}}\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$

$$\frac{\sin 2\theta_2}{\cos 2\theta_2} = \boxed{\tan 2\theta_2 = -\frac{\sigma_x-\sigma_y}{2\tau_{xy}}} \quad \text{--- (5)}$$



(5) te bulunan açılar (2) de yerine yazılırsa; τ_{max} ve τ_{min} i bulunur

$$\tau_{\substack{max\\mini}} = -\frac{\sigma_x-\sigma_y}{2}\cdot\underbrace{\frac{\left(\pm\frac{\sigma_x-\sigma_y}{2}\right)}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}}_{\sin 2\theta_2} + \tau_{xy}\underbrace{\frac{\pm\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}}_{\cos 2\theta_2}$$

$$\boxed{\tau_{\substack{max\\mini}} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}} \quad \text{--- (6)}$$

$$\boxed{\tau_{\substack{max\\mini}} = \pm R}$$

Kayma gerilmelerinin maksimum ve minimum olduğu düzlemlerde, normal gerilmelerin genel olarak sıfırdan farklı değer aldığını göstermek için (5) açıları, (1) de yerine konursa bu düzlemlerde normal gerilmelerin sıfır olmayıp

$$\boxed{\sigma' = \frac{\sigma_x+\sigma_y}{2}} \quad \text{---} (7)$$

değerine eşit olduğu görülür.

Bu ifade; $\sigma_x+\sigma_y \neq 0$ olduğu sürece; kayma gerilmelerinin max ve mini olduğu düzlemlerde normal gerilmelerinde olacağını gösterir.

Kayma gerilmeli hal için, birbirine dik herhangi iki düzlemde etkiyen normal gerilmelerin toplamının asal gerilmeler toplamına eşit olduğunu

$$\boxed{\sigma_x+\sigma_y=\sigma_1+\sigma_2=\sigma_{n_1}+\sigma_{n_2}} \quad \text{--} (8)$$

ve böyle düzlem çiftlerine etkiyen kayma gerilmelerinin birbirine şiddetçe eşit, işaretçe ters değerler aldığı ispatlanabilir.

Bunun için (4) ifadesinden $(\sigma_1+\sigma_2)$ toplamını oluşturmak ve (2) ifadesinde θ yerine bir kez $\theta_1=\theta$ ve bir kez de $\theta_2=\theta+90^\circ$ koyup sonuçları ~~kay~~ karşılaştırmak yeterlidir.

MOHR GERİLME DAİRESİ DENKLEMİ ELDESİ ve çizimi:

(1) den: $$\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)^2$$

(2) den: $$(\tau_n)^2 = \left(-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2$$

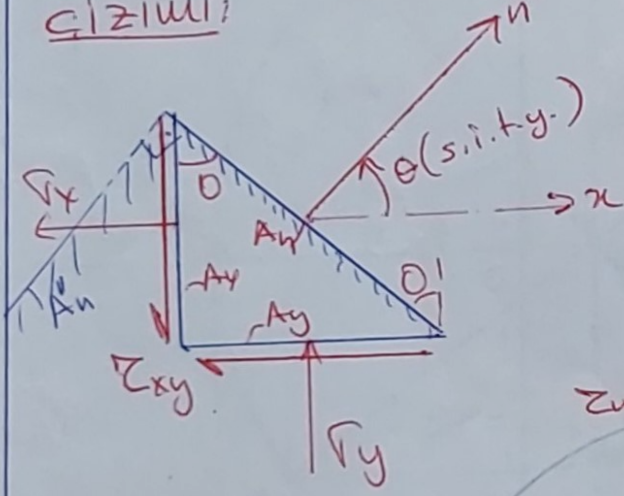
$+$

$$\boxed{\underbrace{\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2}_{x^2} + \underbrace{\tau_n^2}_{y^2} = \underbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}_{R^2}} \quad \text{----(9)}$$

MOHR DAİRESİ DENKLEMİ

$M\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$ $\qquad R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

çizimi:



$A_x(\sigma_x ; \tau_{xy})$

$A_y(-\sigma_y ; -\tau_{xy})$

ölç: $1b = ... N/cm^2$

$\sigma_1 \Rightarrow (2\theta_1)$

$\sigma_2 \Rightarrow (2\theta_1+180°)$

$\tau_{max} = 2\theta_2$

$\tau_{min} = 2\theta_2+180°$

$A_n \perp A'_n$